Kähler–Weyl structures

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Let (M, g, J) be a Hermitian manifold of dimension m; here J is an integrable complex structure on M with $Jg = g$. A *Weyl* structure on (M, g) is a torsion free connection ∇ such that $\nabla g = -2\phi \otimes g$ for some smooth 1-form ϕ. We say that (M, g, J, ∇) is *Kähler–Weyl* if in addition $\nabla J = 0$. Such structures are of interest only in dimension 4 as if $m \geq 6$, then (M, g, J) is (locally) conformally equivalent to a Kähler manifold $(M, \tilde{g}, J)$ so that ∇ is the Levi-Civita connection of $\tilde{g}$. By contrast, in dimension 4, every Hermitian manifold (M, g, J) admits a unique Kähler–Weyl structure where the associated 1-form ϕ is given by $\phi = -\frac{1}{2} dJ\delta\Omega$.

The curvature tensor of a 4-dimensional Kähler–Weyl tensor belongs to a certain representation space Ξ of the unitary group. We show that every algebraic possibility is geometrically representable. The alternating Ricci tensor ρ_a is of particular interest as the Kähler–Weyl structure is trivial if and only if $\rho_a = 0$. We show every algebraic possibility for ρ_a is geometrically represented by a homogeneous example – in particular by a 4-dimensional Lie group with left invariant structures. Similar questions are examined in the pseudo-Hermitian setting and the para-Hermitian setting for signature $(2, 2)$.

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