Extremals For The Sobolev-Folland-Stein Inequality On The Quaternionic Heisenberg Qroup, The Qc Yamabe Problem And Related Geometric Structures

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We describe explicitly non-negative extremals for the Sobolev inequality on the quaternionic Heisenberg groups and determine the best constant in the L^2 Folland-Stein embedding theorem involving quaternionic contact geometry and the quaternionic contact Yamabe equation. Translating the problem to the 3-sasakian sphere, we determine the quaternionic contact Yamabe invariant on the 4n+3-dimensional spheres. In dimension seven, we describe explicitly all solutions to the qc Yamabe equation on the seven dimensional quaternionic Heisenberg group. The main tool is the notion of quaternionic contact structure and the Biquard connection. We show that the torsion of the Biquard connection is an obstruction quaternionic contact structure to be locally isomorphic to a 3-Sasakian one. We define a curvature-type tensor invariant called quaternionic contact (qc) conformal curvature in terms of the curvature and torsion of the Biquard connection. We show that a quaternionic contact manifold is locally qc conformal (gauge equivalent) to the standard flat quaternionic contact structure on the quaternionic Heisenberg group if and only if the qc conformal curvature vanishes. It seems that this tensor could help to prove that the quaternionic contact Yamabe invariant is less or equal to that of the sphere whenever the qc conformal curvature is different or equal to zero which reduces the qc Yamabe problem to that of the spherical qc manifolds.