

Abstract We report on joint work with M. Brozos-Vázquez, E. García-Río, and J. H. Park. The homogeneous affine surfaces have been classified by Opzoda. They may be grouped into 3 families, which are not disjoint. The connections which arise as the Levi-Civita connection of a surface with a metric of constant Gauss curvature form one family (Type \mathcal{C}); there are, however, two other families, one with constant Christoffel symbols on \mathbb{R}^2 (Type \mathcal{A}) and one with Christoffel symbols $\Gamma_{ij}^k := (x^1)^{-1}C_{ij}^k$ on $\mathbb{R}^+ \times \mathbb{R}$ and C_{ij}^k constant (Type \mathcal{B}). The rank of the Ricci tensor plays a central role in our analysis. If the Ricci tensor of a Type \mathcal{A} affine surface is non-singular, we write down a complete set of invariants that determine the local isomorphism type and examine the structure of the associated moduli space. We show the moduli space of Type \mathcal{B} surfaces which are not Type \mathcal{A} and which are not flat is a simply connected real analytic 4-dimensional manifold with second Betti number equal to 1. The higher dimensional setting is quite different and we present some results on Type \mathcal{A} surfaces of dimension $m \geq 3$. In a different direction, the results of Opzoda have been extended to the case where torsion is permitted by Arias-Marco and Kowalski and we discuss some preliminary result in the setting of homogeneous surfaces with torsion.