

PL-STABILITY, FINITENESS AND DIMENSION 4

Frederick Wilhelm

(University of California, Riverside)

JOINT WORK WITH GREG CHAMBERS AND CURTIS PRO

Let $\mathcal{M}_{k,v,d}^{K,V,D}(n)$ denote the class of closed Riemannian n -manifolds M with

$$\begin{aligned} k &\leq \sec M \leq K, \\ v &\leq \text{vol } M \leq V, \text{ and} \\ d &\leq \text{diam } M \leq D, \end{aligned}$$

Diffeomorphism Stability Question. *Given $k \in \mathbb{R}$, $v, D > 0$, and $n \in \mathbb{N}$, let $\{M_\alpha\}_{\alpha=1}^\infty \subset \mathcal{M}_{k,v,0}^{\infty,\infty,D}(n)$ be a Gromov–Hausdorff convergent sequence. Are all but finitely many of the M_α s diffeomorphic to each other?*

I will discuss on going project with Greg Chambers and Curtis Pro in which we prove that the answer to the analogous question for Piecewise Linear Stability is “yes”. In dimensions ≤ 6 , PL-homeomorphic manifolds are diffeomorphic, so this implies that the answer to the Diffeomorphism Stability Question is “yes” provided $n \leq 6$.

Together with Gromov’s precompactness theorem and the Grove-Petersen-Wu finiteness theorem, this yields the following finiteness theorem, which was previously known in all dimensions other than 4.

Theorem: *For any $k \in \mathbb{R}$, $v > 0$, $D > 0$, and $n \in \mathbb{N}$ the class $\mathcal{M}_{k,v,0}^{\infty,\infty,D}(n)$ contains only finitely many diffeomorphism types.*

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Lugar: Aula 9 da Facultade de Matemáticas, USC.

Hora: 16:00 h