Geodesics on standard stationary spacetimes and Lagrangian systems

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Abstract

We present some results concerning a fundamental correspondence between geodesics in standard stationary spacetimes and classical Lagrangian systems.

More precisely, if $(L, \langle\cdot,\cdot\rangle_L)$, $L = M \times \mathbf{R}$, is a standard stationary spacetime and $z = (x, t)$ is a geodesic on L, then the spatial component x of z solves the equation of a classical particle moving on M (endowed with a positive perturbation of its original Riemannian metric) accelerated both by a potential and a magnetic field. The new metric on M, the potential and the magnetic field depend on the coefficients of $\langle\cdot,\cdot\rangle_L$. The energy of x (as solution of the above described Riemannian equation) is equal to the "energy" $1/2\langle\dot{z},\dot{z}\rangle_L$ of the geodesic z.

Thus this correspondence provides a good tool for studying geodesics with fixed energy on this class of spacetimes by variational methods. Indeed, Riemannian solutions with fixed energy (joining two points or periodic) can be found by means a suitable generalization of the classical Maupertuis–Jacobi principle. As a consequence, we obtain existence and multiplicity results for Lorentzian periodic trajectories and geodesics joining a point to a line. We also compare them to previous results on this topic.