Unicity of Constant Higher Order Mean Curvature Spacelike Hypersurfaces in General Robertson-Walker Spacetimes

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Abstract

Consider the following question: Under what conditions must a compact spacelike hypersurface of constant higher order mean curvature in a Lorentzian warped product be a spacelike slice? In collaboration with L. J. Alías we obtained the following result:

Let $-I \times_f M^n$ be a spatially closed Lorentzian warped product obeying the strong null convergence condition, with $n \geq 3$, that is, $K_M \geq sup(ff'' - f'^2)$, where K_M stands for the sectional curvature of M^n. Assume that $\sum^n$ is a compact spacelike hypersurface immersed into $-I \times_f M^n$ which is contained in a slab $\Omega(t_1, t_2) = (t_1, t_2) \times M^n$ on which f' does not vanish. If H_k is constant, with $2 \leq k \leq n$ then $\sum^n$ is totally umbilical. Moreover, $\sum^n$ must be a slice $t_0 \times M^n$ (necessarily with $f'(t_0) \neq 0$), unless in the case where $-I \times_f M^n$ has positive constant sectional curvature and $\sum^n$ is a round umbilical hypersphere.

The proof is based strongly on the ellipticity of second order differential operators L_k associated to higher order mean curvatures of $\sum^n$. (The work will appear in Math. Proceed. Cambridge Philos. Soc.)