Lorentzian metrics with prescribed scalar curvature

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Abstract

In analogy to the prescribed scalar curvature problem in Riemannian geometry, one can ask for a given real-valued function f on a smooth manifold M (which is assumed to admit a Lorentzian metric) whether f is the scalar curvature of some Lorentzian metric g on M. Moreover, one might want to prescribe in addition the connected component of the space of Lorentzian metrics on M in which g should lie. Or, even stronger, one might want to prescribe a field H of tangent hyperplanes on M and demand that g makes each of these hyperplanes spacelike.

We discuss all these problems and show that a suitable metric g exists in most cases. The main tool in the proofs is an elliptic equation which generalises the well-known Yamabe equation from the Riemannian case of the prescribed scalar curvature problem.