Homogeneous Lorentzian Manifolds Of Semisimple Lie Groups

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We discuss the problem of description of homogeneous Lorentzian manifolds $M = G/H$ with (mostly proper) action of an isometry group G and connected stability subgroup H.

We recall the classical result by Nadine Kowalsky about nonproper homogeneous Lorentzian manifolds $M = G/H$ (such that the stabilizer H is non compact) and its recent generalizations by M. Deffaf, K. Melnick and A. Zeghib.

We give a necessary and sufficient condition that a proper homogeneous manifold $M = G/H$ admits an invariant Lorentz metric. A homogeneous manifold $M = G/H$ with a compact stabilizer H is called minimal admissible if it admits an invariant Lorentzian metric, but any homogeneous manifold $\tilde{M} = G/\tilde{H}$ with a bigger (connected) stability group $\tilde{H} \supset H$ does not admits such a metric.

We classify minimal admissible compact homogeneous manifolds $M = G/H$. In particular, if the group G is simple, any such manifold is a circle bundle over a minimal adjoint orbit of the group G. We reduce the classification of minimal admissible manifold $M = G/H$ of a simple non-compact Lie group G to description of minimal orbits of the isotropy representation of the associated non-compact symmetric space $S = G/K$ and give a list of such manifolds of dimension ≤ 11.

We discuss also the problem of description of nonproper homogeneous Lorentzian manifolds of non semisimple Lie group G and determine such manifolds with irreducible action of the isotropy group on the screen space.