The Lorentzian Conformal Analogon Of Calabi-Yau Manifolds

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Calabi-Yau manifolds are Riemannian manifolds with holonomy group SU(m). They are Ricci-flat and Kähler and admit a 2-parameter family of parallel spinors. In the talk we will discuss the Lorentzian conformal analogue of this situation. If on a manifold a class of conformally equivalent metrics [g] is given, then one can consider the holonomy group of the conformal manifold (M; [g]), which is a subgroup of O(p + 1; q + 1) if the metric g has signature (p; q). There is a close relation between algebraic properties of the conformal holonomy group and the existence of Einstein metrics in the conformal class as well as to the existence of conformal holonomy groups of Lorentzian manifolds. In particular, we will describe Lorentzian manifolds (M; g) with conformal holonomy group SU(1; m), which can be viewed as the conformal analogon of Calabi-Yau manifolds. Such Lorentzian metrics g, known as Fefferman metrics, appear on S^1 -bundles over strictly pseudoconvex CR spin manifolds and admit a 2-parameter family of conformal Killing spinors.