Actions Of Cohomogeneity One On Symmetric Spaces Of Noncompact Type

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This is joint work with Hiroshi Tamaru (Hiroshima University). An isometric action of a Lie group on a Riemannian manifold is of cohomogeneity one if the principal orbits have codimension one. If the Riemannian manifold is a symmetric space of noncompact type, then either the orbits form a Riemannian foliation, or there exists exactly one singular orbit. In [BT03] we classified the cohomogeneity one actions whose orbits form a foliation, and in [BT04] we classified those with a totally geodesic singular orbit. However, there are many cohomogeneity one actions with a non-totally geodesic singular orbit, as we have shown in [BT07] for the rank one case. In the recent work [BT10] we developed a conceptual approach for classifying the actions with a non-totally geodesic singular orbit. Geometrically this approach is based on the horospherical decomposition of the symmetric space, algebraically it is based on the Langlands decomposition of a real semisimple Lie algebra. The main result states that there are two construction methods for such cohomogeneity one actions. Roughly speaking, the first method is a canonical extension process using cohomogeneity one actions on certain totally geodesic submanifolds of the symmetric space, and therefore leads to a rank reduction procedure for the classification. The second type is more involved and not yet fully understood. In the talk I will give a brief survey about the known results, and then explain the conceptual approach and discuss several examples. I will also briefly explain what the remaining open problems are for achieving a full classification.

References

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