Principal Curvatures Of Isoparametric Hypersurfaces In Complex Hyperbolic Spaces

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In any Riemannian ambient manifold, a hypersurface is called isoparametric if it and its sufficiently close equidistant hypersurfaces have constant mean curvature. An (extrinsically) homogeneous hypersurface is a codimension one orbit of an isometric action. It is easy to show that every homogeneous hypersurface is isoparametric and has constant principal curvatures. However, the converse is not true in general.

In this talk, we study isoparametric hypersurfaces in nonflat complex space forms and, in particular, in complex hyperbolic spaces.

We show that the number of principal curvatures and their multiplicities of an isoparametric hypersurface in a complex hyperbolic space are basically the same as those of the homogeneous examples. The number h of nontrivial projections of the Hopf vector field onto the principal curvature spaces is then bounded by 3. If $h \leq 2$ then the hypersurface has constant principal curvatures, and we can prove that it is homogeneous. For $h = 3$ we construct inhomogeneous isoparametric hypersurfaces with nonconstant principal curvatures.