Minimal Vector Fields On Riemannian Manifolds

Olga Gil

Universidad de Valencia, Spain

We consider a vector field V defined on a manifold M, of dimension n, as being a map from M into the tangent manifold TM. Given a Riemannian metric g on M, there is a natural metric on TM, known as the Sasaki metric. The submanifold $V(M) \subset TM$, diffeomorphic to M, is endowed with the corresponding induced metric and we can consider the n-dimensional volume of this submanifold. It is easy to see that if the vector field V is parallel, with respect to the Levi-Civita connection of the metric g, then $V(M)$ is a totally geodesic submanifold of TM, isometric to (M, g), that has the minimum volume among all submanifolds obtained from smooth vector fields of the same lenght that V. If the Riemannian manifold (M, g) does not admit parallel vector fields, a natural question is to compute the value of the infimum of the volume of smooth unit vector fields and to find those vector fields determining submanifolds of TM of less possible volume. This is known as the Gluck and Ziller problem; it was proposed in a basic paper of these authors in 1986 and it is far for being solved.