On Curvature Homogeneous Spaces And Their Modifications

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The story started with an old paper by prof. K. Sekigawa (Hokkaido Math. Journal, 1973). We will survey some older results (with E. Boeckx, F. Tricerri, L. Vanhecke) and some very fresh results inspired by this remarkable paper (with Z. Dušek and A. Vanžurová). For example, an older result says that every irreducible Riemannian manifold with the same curvature tensor as a symmetric space and NOT locally homogeneous (so-called "non-homogeneous relative of a symmetric space") is a "generalized Sekigawa example". One recent result offers a pseudo-Riemannian version of the generalized Sekigawa examples - a family of irreducible and not locally homogeneous pseudo-Riemannian spaces with arbitrary signature (p,q) whose model space is a symmetric pseudo-Riemannian space either of the form $M^2(c) \times \mathbb{R}^{p-2}_q$, or of the form $L^2(c) \times \mathbb{R}^{p-1}_{q-1}$, where $M^2(c)$ is a 2-dimensional Riemannian space form, $L^2(c)$ a 2-dimensional Lorentzian space form, and $\mathbb{R}^k_l$ denotes a pseudo-Euclidean space with signature (k, l). The curvature homogeneous pseudo-Riemannian manifolds with general signature (also in higher orders) were studied profoundly by P. Gilkey and his collaborators. Yet, it seems that they did not pay much attention to spaces with a symmetric model. Another topic is introducing a new concept of a curvature-homogeneous space, namely in the sense that not the curvature tensor of type $(0, 4)$ is pre- served from point to point but the curvature tensor of type $(1, 3)$ is preserved from point to point, in some sense. Each curvature-homogeneous space in the classical sense is also curvature-homogeneous in the modified sense. We first give proper examples of the new spaces in all dimensions. Further, a complete classification of such spaces in dimension 3 and in generic case is given. Finally, we show that the original example by K. Sekigawa (which is curvature homogeneous just of order zero) is curvature homogeneous up to order one in the modified sense.