

ABOUT THE LORENTZIAN VERSION OF CLASSICAL NASH'S THEOREM ON ISOMETRIC EMBEDDINGS

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A celebrated theorem by J. Nash states that any smooth Riemannian manifold is (globally) isometrically embeddable in some R^N . From this theorem, one can check that any semi-Riemannian manifold can be isometrically embedded in a semi-Euclidean space R_s^N of sufficiently big index s and dimension N . However, not all the Lorentzian manifolds can be isometrically embedded in Lorentz-Minkowski L^N . A conjecture which goes back to the work by CJS Clarke (1970), states that any globally hyperbolic spacetimes is isometrically embeddable.

In this talk, I will explain the following positive answer to this conjecture, obtained in collaboration with O. Müller (arXiv:0812.4439v4). First, the Lorentzian manifolds isometrically embeddable in L^N are characterized as those spacetimes which admit a time function τ with $|\tau| > 1$. Then, globally hyperbolic spacetimes are shown to admit such a function τ . For this second part, the role and difficulties derived from the so-called *folk problems of smoothability*, are stressed.