A Curvature Identity On A 4-Dimensional Riemannian Manifold And Its Applications

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Berger discussed the generalized Gauss-Bonnet formula on a 4-dimensional compact oriented Riemannian manifold $M = (M, g)$ from the variational theoretic view point and derived a curvature identity on M. Further, he gave a negative answer to the problem raised by Singer. Recently, Labbi extended Berger's result to higher dimensional cases by using an elegant method.
First, we shall prove that the curvature identity obtained by Berger holds on any 4-dimensional Riemannian manifold which is not necessarily compact. Further, we may also check that this result can be extended to the pseudo-Riemannian case with the aid of computer support program such as *Mathematica*. Next, we shall define a *weakly Einstein manifold* as a generalization of a 4-dimensional Einstein manifold based on the curvature identity, and discuss the fundamental properties of such manifolds in connection with the Einstein case.

References

[1] M. Berger, Quelques formues de variation pour une structure riemannienne, Ann. Sci. Econ. Norm. Sup.**3**(4)(1970), 285-294.

[2] Y. Euh, J. H. Park and K. Sekigawa, A curvature identity on a 4-dimensional Riemannian manifold, preprint.

[3] Y. Euh, J. H. Park and K. Sekigawa, A generalization of a 4-dimensional Einstein manifolds, preprint.

[4] O. Kowalski, On the Gauss-Kronecker curvature tensors, Math. Ann. **203**(1973), 335-343.

[5] M.-L. Labbi, Variational properties of the Gauss-Bonnet curvatures, Calc. Var. **32**(2008), 175-189.