Geometric Modelling Of The Solution Of Some Monge-Ampère Equations

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In the book [1] we give a proof of the following extension of the famous Theorem of Jörgens-Calabi-Pogorelov in any dimension:

Let $u : \mathbb{R}^n \to \mathbb{R}$, $(\xi_1, \dots, \xi_n) \mapsto u(\xi_1, \dots, \xi_n)$, be a strictly convex C^∞-function. If u satisfies the PDE of Monge-Ampère type

$$\det \left(\frac{\partial^2 u}{\partial \xi_i \partial \xi_j} \right) = \exp \left\{ - \sum c_i \frac{\partial u}{\partial \xi_i} - c_0 \right\}$$

where c_0, c_1,..., c_n are real constants, then u must be a quadratic polynomial. The proof of this extended Theorem is very hard in arbitrary dimension n, see [1], it is easier for $n \leq 4$.

We sketch the proof for dimension $n \leq 4$ and compare this proof with one for the Theorem of Jörgens-Calabi-Pogorelov; the comparison admits to survey a geometric modelling procedure for solutions of certain Monge-Amp‘ere equations.

References

[1] A. M. Li, R. Xu, U. Simon, F. Jia, Affine Bernstein Problems and Monge-Ampère equations, World Scientific, 2010.