POSTER SESSION A (MONDAY 13)

Contents

Some Remarks On The Gaussian Curvature Of Spacelike Surfaces In Lorentzian Product Spaces

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Given Σ^2 a compact spacelike surface immersed in a Lorentzian product space $M^2 \times \mathbb{R}_1$, we establish an integral formula which allows us to derive some interesting consequences in terms of the Gaussian curvature of the surface. For instance, when M^2 is either the sphere $\mathbb{S}^2$ or the real projective plane $\mathbb{RP}^2$, we characterize the slices of the trivial totally geodesic foliation $M^2 \times \{t\}$ as the only complete spacelike surfaces with constant Gaussian curvature in the Lorentzian product $M^2 \times \mathbb{R}_1$. On the other hand, we show that our results are no longer true when $M^2 = \mathbb{H}^2$ is the hyperbolic plane. In fact, we give examples of complete spacelike surfaces with constant Gaussian curvature $K \leq -1$. However, using the abstract theory of Codazzi pairs, we show that there exists no complete spacelike surface in $\mathbb{H}^2 \times \mathbb{R}_1$ with constant Gaussian curvature $K > -1$.

Compact Indefinite Globally Framed f-Manifolds And Toroidal Principal Bundles

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It is well-known that the language of principal bundles furnishes the needed elements in Gauge Theory to describe the interaction of a particle, seen as an object of the base space, with a gauge field, that is the curvature 2-form Ω of a connection ω on a smooth principal G-bundles $G \hookrightarrow P \to X$. Moreover, further attention may be reserved for those principal bundles whose total and/or base spaces are smooth manifolds endowed with a Lorentz metric compatible with special geometric structures. We are interested in globally framed f-manifolds ($g.f.f$-manifolds for short). In particular manifolds known as Lorentz framed manifolds ([3]), may be used as spacetime manifolds in General Relativity (see also [4]).

Studies on toroidal principal bundles have been started in the Riemannian setting by Blair, Ludden, Yano, Morimoto et al. (cf. for example [1, 2, 7]), giving the fundamental relationships between f-manifolds and Riemannian submersions. Here we are concerned with some extensions of their results to toroidal principal bundles in the semi-Riemannian context, and this involves in a natural way semi-RiemanciteONArt,O'N) with totally geodesic fibres. These objects have their own well-known physical importance (see [5], or last chapter of [6]), since a (semi-)Riemannian submersion with totally geodesic fibres is a harmonic map and a harmonic map is known by the physicists as σ-model.

About the extensions of the results of [2] to the semi-Riemannian case, we consider a connected, compact smooth manifold M endowed with a normal indefinite $g.f.f$-structure. Under suitable hypotheses of regularity, we get that M is the total space of a toroidal principal bundle and the structure projects over an (indefinite) Kähler structure or an (indefinite) Sasakian one. On the other hand, we show that the total space of a toroidal principal bundle over a Kähler manifold, indefinite or not, may admit indefinite metrics, and the lift of the (indefinite) Kähler structure gives rise to normal $g.f.f$-structures on the total space.

We give some applications, in particular we construct a Lorentzian $\mathcal{S}$-structure on the compact Lie group $U(2)$ having two characteristic vector fields with different causal character and we obtain three quotient manifolds of $U(2)$ linked by semi-Riemannian submersions in a commutative diagram. We also prove that $U(2)$ with such a structure is foliated by Reinhart lightlike hypersurfaces.

References

[1] Blair,D.E.: *Geometry of manifolds with structural group $\mathcal{U}(n) \times \mathcal{O}(s)$.* J. Differential Geom. 4, 155–167 (1970).

[2] Blair, D.E., Ludden, G.D., Yano, K.: *Differential geometric structures on principal toroidal bundles.* Trans. Amer. Math. Soc. 181, 175–184 (1973).

[3] Duggal, K.L.: *Lorentzian geometry of globally framed manifolds.* Acta Appl. Math. 19, 131–148 (1990).

[4] Duggal, K.L., Sharma, R.: *Symmetries of Spacetimes and Riemannian Manifolds.* Kluwer Academic Publishers, Dordrecht Vol. 487 (1999).

[5] Falcitelli, M., Ianuş, S., Pastore, A.M., Vişinescu, M.: *Some applications of Riemannian submersions in Physics.* Rev. Roum. Phys. 48, 627–639 (2003).

[6] Falcitelli, M., Ianus, S., Pastore, A.M.: *Riemannian submersions and related topics.* World Sci. Publishing, River Edge, NJ (2004).

[7] Morimoto, A.: *On normal almost contact structures with a regularity.* Tôhoku Math. J. 2 16, 90–104 (1964).

[8] O'Neill, B.: *The fundamental equations of a submersion.* Michigan Math. J. 13, 459–469 (1966).

[9] O'Neill, B.: *Semi-Riemannian geometry.* Academic Press, New York (1983).

Calabi-Bernstein Type Problems For Certain Generalized Robertson-Walker Spacetimes

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Uniqueness and non-existence results of entire solutions to the maximal surface equation and to the constant mean curvature spacelike surface equation in certain Generalized Robertson-Walker spacetimes obeying several energy conditions are found. Motivated by this problem we study the corresponding parametric version.

Osserman Metrics And Riemannian Extensions

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(joint work with Eduardo García-Río, Peter Gilkey and Ramón Vázquez-Lorenzo)

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The Riemannian extension of a torsion-free affine manifold (M, D) is a pseudo-Riemannian metric on the cotangent bundle T^*M whose curvature reflects many properties of the affine structure. On the other hand some modifications of the classical Riemannian extensions are the natural structure underlying certain geometries like the paracomplex space forms, which correspond to modified Riemannian extensions of flat connections.

Riemannian extensions of torsion-free connections with nilpotent Jacobi operators (i.e., affine Osserman) produce examples of Osserman manifolds with nilpotent Jacobi operators. Here we show how modified Riemannian extensions of affine Osserman manifolds give rise to new examples of Osserman manifolds whose Jacobi operators are neither diagonalizable nor nilpotent.

Natural Diagonal Para-Kähler Tangent Bundles

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We characterize the natural diagonal almost product (locally product) structures on the tangent bundle of a Riemannian manifold. We obtain the conditions under which the tangent bundle endowed with the determined structure and with a metric of natural diagonal lift type is a Riemannian almost product (locally product) manifold, or an (almost) para-Hermitian manifold. Finally, we find the natural diagonal (almost) para-Kählerian structures on the tangent bundle.

On The Behavior Of The Scalar Curvature Of Constant Mean Curvature Hypersurfaces In Space Forms

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(joint work with Luis J. Alías)

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In this work we study the behavior of the scalar curvature Scal of a hypersurface Σ immersed with constant mean curvature into a Riemannian space form of constant curvature, deriving a sharp estimate for the infimum and for the supremum of Scal, under some appropriate conditions on Σ, such as properly immersed and stochastic completeness. Our results will be an application of a generalized Omori-Yau maximum principle due to Pigola, Rigoli and Setti.

Keywords: constant mean curvature, scalar curvature, Ricci curvature, maximum principle, stochastic completeness.

Some Examples Of Lorentzian Ricci Solitons

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We study different families of Lorentzian Ricci solitons, each of them is related to some specific geometric aspect. First we provide a complete classification of 3-dimensional Lorentzian homogeneous Ricci solitons, showing that all non-trivial examples have non-diagonalizable Ricci operator with three equal eigenvalues. Secondly, we show that Lorentzian manifolds whose isometry group is of dimension at least $\frac{1}{2}n(n-1)+1$ are expanding, steady and shrinking Ricci solitons. Moreover these are also gradient Ricci solitons of steady type.

Seiberg-Witten Equations On Pseudo-Riemannian $Spin^c$ Manifolds With Neutral Signature

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(joint work with Nedim Değirmenci)

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We consider pseudo-Riemannian $4-$manifolds with neutral signature whose structure group are $SO_+(2,2)$. We prove that such manifolds have pseudo riemannian *spin*^{c} structure. We construct spinor bundles S, S^+, S^- on these manifolds. For the first Seiberg-Witten equation we define Dirac operator on these bundles. Due to the neutral metric self-duality of a $2-$form is meaningful and it enable us to write down second Seiberg-Witten equation. Lastly we write explicit forms of these equation on $4-$dimensional flat space.

Conformal Relatives Of Symmetric Spaces

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A Riemannian manifold is called *curvature homogeneous*, if its curvature tensor at any two points is "the same". A curvature homogeneous manifold is *modelled on a homogeneous space* M_0, if its curvature tensor at every point is "the same" as the curvature tensor of M_0. A curvature homogeneous manifold modelled on a symmetric space M_0 is locally isometric to it, unless the de Rham decomposition of M_0 contains a product of a Euclidean space and a two-space of a constant nonzero curvature (Kowalski, Tricerri, Vanhecke 1992).

Likewise, a Riemannian manifold is called *Weyl homogeneous*, if its Weyl conformal curvature tensor at any two points is "the same", up to a positive multiple. A Weyl homogeneous manifold is *modelled on a homogeneous space* M_0, if its Weyl tensor at every point is "the same" as the Weyl tensor of M_0, up to a positive multiple.

We prove that a Weyl homogeneous manifold M^n, $n \geq 4$, modelled on a symmetric space M_0 is conformally equivalent to M_0 in each of the following cases: the de Rham decomposition of M_0 contains no factors of rank two or less; M_0 is a compact simple Lie group with a bi-invariant metric or on its noncompact dual; M_0 has rank one.

A Moduli Space Of Minimal Affine Lagrangian Submanifolds

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In the paper "*Deformations of calibrated submanifolds*" (Comm. Anal. Geom. (1998)) R. McLean proved that special Lagrangian submanifolds nearby a compact special Lagrangian submanifold of a Calabi-Yau manifold form a manifold of dimension b_1, where b_1 is the first Betti number of the submanifold. Then a few papers giving generalizations to the cases where the ambient space is not a Calabi-Yau manifold but a more general type of space have been published. All those cases lie, in fact, within metric geometry. Moreover these are local results. A similar result in a non-metric case is proposed. Our result is global. We describe the set of all minimal affine Lagrangian embeddings of a compact manifold. To this aim we use the notion of a Fréchet manifold introduced by R. Hamilton in 1982.

It seems that from a differential geometry viewpoint non-metric analogues of Calabi-Yau manifolds are equiaffine complex manifolds, that is, complex manifolds equipped with a torsion-free complex connection and a non-vanishing covariant constant complex volume form. There are many very natural complex equiaffine manifolds. For instance, complex hypersurfaces of the complex affine space $\mathbf{C}^n$ or the tangent bundles of the so call with parallel volume (according to the definition given by L. Auslander). Equiaffine structures are, in general, non-metrizable.

In order to discuss minimality of submanifolds a metric structure is not necessary. It is sufficient to have induced volume elements on submanifolds. Such a situation exists in case of affine Lagrangian submanifolds. A real n-dimensional submanifold of a complex n-dimensional manifold is affine Lagrangian if the tangent bundle of the submanifold is transversal to its image by the complex structure. An oriented affine Lagrangian submanifold of a complex equiaffine manifold is naturally endowed with an induced volume form. Such a submanifold (if connected) is minimal iff its phase function is constant. This condition is directly related with the notion of calibrations introduced by Harvey and Lawson (1981). A compact minimal affine Lagrangian submanifold is volume minimizing in its cohomology class. A cohomologous family of minimal submanifolds can be treated as simultaneously calibrated by the real part of the complex volume form in the ambient space.

The following result will be presented

Theorem. Let N be a $2n$-dimensional almost complex manifold equipped with a smooth nowhere-vanishing closed complex n-form Ω. Let M be a connected compact orientable n-dimensional real manifold admitting a minimal (relative to Ω) affine Lagrangian embedding into N.

Let $\mathcal{M}$ denote the Fréchet manifold $\mathcal{C}^{\infty}_{emb}(M,N)_{/Diff^{\infty}(M)}$ and $\mathcal{M}aL$ its open (in the $\mathcal{C}^1$-topology) submanifold determined by affine Lagrangian embeddings. Then

$$\mathcal{M}maL = \{[f] \in \mathcal{M}aL;\ f \ is\ minimal\}$$

is an infinite dimensional manifold modeled on the Fréchet vector space of all closed $(n-1)$-forms of class $\mathbf{C}^{\infty}$ on M. It is a submanifold of $\mathcal{M}aL$.

Hypersurfaces In The Lorentz-Minkowski Space Satisfying $L_k\psi = A\psi + b$

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We study hypersurfaces in Lorentz-Minkowski space $\mathbb{L}^{n+1}$ whose position vector ψ satisfies the condition $L_k\psi = A\psi + b$, where L_k is the linearized operator of the $(k+1)$th mean curvature of the hypersurface for a fixed $k = 0, \dots, n-1$, $A \in \mathbb{R}^{(n+1)\times(n+1)}$ is a constant matrix and $b \in \mathbb{L}^{n+1}$ is a constant vector. For every k, we prove that the only hypersurfaces satisfying that condition are hypersurfaces with zero $(k+1)$th mean curvature, open pieces of totally umbilical hypersurfaces $\mathbb{H}^n_1(r)$ or $\mathbb{H}^n(-r)$, and open pieces of generalized cylinders $\mathbb{S}^m_1(r)\times\mathbb{R}^{n-m}$, $\mathbb{H}^m(-r)\times\mathbb{R}^{n-m}$, with $k+1 \leq m \leq n-1$, or $Ł^m \times §^{n-m}(r)$, with $k+1 \leq n-m \leq n-1$. This completely extends to the Lorentz-Minkowski space a previous classification for hypersurfaces in $\mathbb{R}^{n+1}$ given by Alías and Gürbüz [Geom. Dedicata 121 (2006), 113–127].

Keywords: linearized operator L_k; isoparametric hypersurface; k-maximal hypersurface; Takahashi theorem; higher order mean curvatures; Newton transformations.

Noncommutative Bloch Analysis Of Bochner Laplacians

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Given an invariant Hermitian connection in a vector fiber bundle and a periodic semi-bounded scalar function $\tilde{V}$ on a Riemannian manifold $\tilde{M}$ with a discrete symmetry group Γ, consider the Γ-periodic Hamilton operator $\tilde{H} = -\tilde{\Delta}_{\mathrm{B}} + \tilde{V}$ where $\tilde{\Delta}_{\mathrm{B}}$ is the Bochner Laplacian. The symmetry group Γ can be noncommutative, and the connection need not flat. On the other hand, Γ is supposed to be of type I. With any unitary representation Λ of Γ one associates an operator $H^{\Lambda} = -Delta_{\mathrm{B}}^{\Lambda} + V$ on $M = \tilde{M}/\Gamma$ where V is the projection of $\tilde{V}$ to M. Using the Fourier transformation on Γ we describe a generalized construction of the Bloch decomposition of $\tilde{H}$ into a direct integral whose components are H^{Λ}, with Λ running over the dual space $\hat{\Gamma}$. The evolution operator, the heat operator and the resolvent decompose correspondingly. The Bloch decomposition is known to be an important tool in the spectral analysis of magnetic Schrödinger operators. Conversely, given $\Lambda \in \hat{\Gamma}$, one can express the propagator $\mathcal{K}_t^{\Lambda}(y_1, y_2)$ (= the kernel of $\exp(-itH^{\Lambda})$) in terms of the propagator $\tilde{\mathcal{K}}_t(y_1, y_2)$ (= the kernel of $\exp(-it\tilde{H})$) asr Γ. Again, analogous formulas exist for the corresponding heat kernels and Green functions. Formulas of this type find their application both in mathematics, for example in derivation of the Selberg trace formula, and in theoretical physics, for example in the case when a configuration manifold M is multiply connected and $\tilde{M}$ is its universal covering space.

Slant Immersions Of Quaternionic Space Forms

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In this paper we investigate the existence of quaternionic slant immersions in quaternionic space forms with unfull first normal bundle.