POSTER SESSION B (WEDNESDAY 15)

Contents

On Symbols For Fourier Integral Operators

Mohsen Alimohammady

University of Mazandaran, Iran

In this paper, the symbols will present for some classes of Fourier integral operators and some properties of them will investigate. We will achieve an asymptotically version of this symbols.

About The Determination Of Closed Symmetric-Like Riemannian Manifolds Through The Spectrum Of Laplace-Beltrami Operator

Teresa Arias-Marco

University of Extremadura, Spain

D'Atri spaces, manifolds of type $\mathcal{A}$, probabilistic commutative spaces, $\mathfrak{C}$-spaces, $\mathfrak{TC}$-spaces, and $\mathfrak{GC}$-spaces have been studied by many authors as symmetric-like Riemannian manifolds.

In the poster we study the determination of the previous properties on closed Riemannian manifolds using the eigenvalue spectrum of the associated Laplace operator on functions. Moreover, we will introduce the class of weakly locally symmetric manifolds and studied its determination in the closed case too.

Joint work with Dorothee Schueth.

Some Results About Log-Harmonic Functions

Melike Aydoğan

Yeni Yuzyil University, Turkey

Let $A(\alpha, \beta)$ is a subclass of certain analytic functions and $H(\mathbb{D})$ is to be a linear space of all analytic functions defined on the open unit disc $\mathbb{D} = \{z| \, |z| < 1\}$. A sense-preserving log-harmonic function is the solution of the non-linear elliptic partial differential equation

$$\overline{f_{\bar{z}}} = w \frac{\overline{f}}{f} f_z,$$

where $w(z)$ is analytic, satisfies the condition $|w(z)| < 1$ for every $z \in \mathbb{D}$ and is called the second dilatation of f. It has been shown that if f is a non-vanishing log-harmonic mapping then f can be represented by

$$f(z) = h(z)\overline{g(z)},$$

where $h(z)$ and $g(z)$ are analytic in $\mathbb{D}$ with $h(0) \neq 0$, $g(0) = 1$([**?**]). If f vanishes at $z = 0$ but it is not identically zero, then f admits the representation

$$f(z) = z\,|z|^{2\beta}\,h(z)\overline{g(z)},$$

where $Re\beta > -\frac{1}{2}$, $h(z)$ and $g(z)$ are analytic in D with $g(0) = 1$ and $h(0) \neq 0$. The class of sense-preserving log-harmonic mappins is denoted by $\mathcal{S}_{LH}$. The aim of this paper is to give some distortion theorems of these classes.

Results On Biharmonic Submanifolds In Spheres

A. Balmus

(joint work with C. Oniciuc and S. Montaldo)

Al.I. Cuza University of Iasi, Romania

The study of proper (non-minimal) biharmonic submanifolds in curved spaces represents an important research direction in the theory of biharmonic maps. We shall present old and new classification results for proper biharmonic submanifolds in the unit n-dimensional Euclidean sphere.

Transitions Among $SO(3)/\sim$, S^{2+} And $Q_0/\sim$

Senay Baydas and Bulent Karakas

Yüzüncü Yil University, Turkey

Defining a equivalence relation on $O(3)$ and a new group operation on unit sphere, a group structure is obtained on S^{2+}. Another group structure on S^{2+} was constructed using one-to-one corresponding between S^{2+} and $Q_0/\sim$. The transitions among $SO(3)/\sim$, S^{2+} and $Q_0/\sim$ were constructed. Some examples were given and applied at Matlab.

Harmonic Maps Of Finite Uniton Number Into G_2

N. Correia

(joint work with R. Pacheco)

University of Beira Interior, Portugal

We establish explicit formulae for canonical factorizations of extended solutions corresponding to harmonic maps of finite uniton number into the exceptional Lie group G_2 in terms of the Grassmannian model for $\Omega_{\mathrm{alg}}G_2$, the group of based algebraic loops in G_2. A description of the "Frenet frame data" for such harmonic maps is given. In particular, we show that harmonic spheres into G_2 correspond to solutions of certain algebraic systems of quadratic and cubic equations.

Reference: N. Correia and R. Pacheco, "Harmonic maps of finite uniton number into G_2", arXiv:1007.4477v1 [math.DG]

Principal Curvatures Of Isoparametric Hypersurfaces In Complex Hyperbolic Spaces

Miguel Domínguez-Vázquez

(joint work with J. Carlos Díaz-Ramos)

Universidad de Santiago de Compostela, Spain

We show that the number of principal curvatures and their multiplicities of an isoparametric hypersurface in a complex hyperbolic space are basically the same as those of the homogeneous examples. The number h of nontrivial projections of the Hopf vector field onto the principal curvature spaces is then bounded by 3. If $h \leq 2$ then the hypersurface has constant principal curvatures, and we can prove that it is homogeneous. For $h = 3$ we construct inhomogeneous isoparametric hypersurfaces with nonconstant principal curvatures.

On Some Classes Of N(k)-Quasi Einstein Manifolds

Richard S. Lemence

(joint work with Shyamal Kumar Hui)

Ochanomizu University, Japan

A Riemannian manifold $(M^n, g)(n > 2)$ is said to be quasi Einstein manifold [1] if its Ricci tensor S of type (0, 2) is not identically zero and satisfies the following:

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y), \tag{1}$$

for the associated vector field ξ. The 1-form η is called the associated 1-form and the unit vector field ξ is called the generator of the manifold. An n-dimensional manifold of this kind is denoted by $(QE)_n$. The scalars a and b are known as the associated scalars. In cosmology, if the existence of a 4-dimensional Lorentz manifold is established whose Ricci tensor is of the form (1.1), then such a space-time represents a perfect space-time. In [2], a conformally flat perfect fluid space-time has a geometric structure of a space of quasi-constant curvature, a natural sub-class of quasi Einstein manifold. The study of quasi Einstein manifolds helps us to have a deeper understanding of the global character of the universe including its topology [3]. As a consequence, we can investigate the nature of singularities defined from a differential geometric standpoint.

Tanno, in [15], gave the k-nullity distribution of a Riemannian manifold. The k-nullity distribution $N(k)$ of a Riemannian manifold M is defined by

$$N(k) : p \to N_p(k) = \{U \in T_pM : R(X,Y)U = k[g(Y,U)X - g(X,U)Y]\} \tag{2}$$

for all $X, Y \in TM$, where k is some smooth function and R is the curvature tensor.

The notion of $N(k)$-quasi Einstein manifold was introduced by Tripathi and Kim [16]. If the generator ξ of a quasi Einstein manifold belongs to the k-nullity distribution $N(k)$ for some smooth function k, then this quasi Einstein manifold is called $N(k)$-quasi Einstein manifold [16]. The $N(k)$-quasi Einstein manifolds has also been studied by Özgür [8], Özgür and Sular [9], Özgür and Tripathi [11].

In 1971, Pokhariyal and Mishra [13] introduced new tensor fields, called W_2 and E tensor fields, in a Riemannian manifold and studied their properties. According to them a W_2-curvature tensor on a manifold $(M^n, g)(n > 3)$ is defined by

$$W_2(X,Y)Z = R(X,Y)Z + \frac{1}{n-1}[g(X,Z)QY - g(Y,Z)QX], \tag{3}$$

where Q is the Ricci operator, i.e., $g(QX,Y) = S(X,Y)$ for all X, Y. Here, we studied the W_2-curvature tensor field of an $N(k)$-quasi Einstein manifold and generalized Ricci recurrent $N(k)$-quasi Einstein manifolds.

A Riemannian manifold M is locally symmetric if its curvature tensor R satisfies $\nabla R = 0$, where ∇ is Levi-Civita connection of the Riemannian metric. The notion of locally symmetric manifold has been weakened by many authors in several ways. As a proper generalization of locally symmetric manifolds, the notion of semisymmetric manifolds was defined in [14] by $R(X,Y) \cdot R = 0$. We studied W_2-semisymmetric and W_2-symmetric $N(k)$-quasi Einstein manifolds and it is shown that there exist no W_2-semisymmetric $N(k)$-quasi Einstein manifold. However, it is proved that there exist W_2-symmetric $N(k)$-quasi Einstein manifold, provided that the associated scalar b is non-zero constant. Moreover, it is shown that in an $n(>3)$-dimensional W_2-conservative $N(k)$-quasi Einstein manifold with b is non-zero constant, the associated 1-form η is closed and the integral curves of the generator ξ are geodesics.

The notion of generalized Ricci recurrent manifolds was introduced by De, Guha and Kamilya [4]. We also studied the generalized Ricci recurrent $N(k)$-quasi Einstein manifolds. It is well known that a locally symmetric manifold is Ricci parallel and the converse holds for dimension three. By the decomposition of the covariant derivative ∇S of the Ricci tensor S of type (0, 2), Gray [6] introduced two important classes $\mathcal{A}$, $\mathcal{B}$, which lie between the class of Ricci parallel manifolds and the manifolds of constant curvature; namely, (i) the class $\mathcal{A}$ is the class of manifolds whose Ricci tensor is cyclic parallel and (ii) the class $\mathcal{B}$ is the class of manifolds whose Ricci tensor is of Codazzi type. Both the classes of generalized Ricci recurrent $N(k)$-quasi Einstein manifolds are classified. Finally, the existence of $N(k)$-quasi Einstein manifold is ensured by some non-trivial examples.

References

[1] Chaki, M. C. and Maity, R. K., *On quasi Einstein manifolds*, Publ. Math. Debrecen, **57 / 3-4** (2000), 297–306.

[2] Chaki, M. C. and Ghosh, M. L., *On quasi Einstein manifolds*, Ind. Jour. Maths., **42** (2000), 211–220.

[3] Chaki, M. C. and Ghoshal, P. K., *Some global properties of quasi Einstein manifolds*, Publ. Math. Debrecen, **63 / 4** (2003), 635–641.

[4] De, U. C., Guha, N. and Kamilya, D., *On generalized Ricci recurrent manifolds*, Tensor, N. S., **56** (1995), 312–317.

[5] Ferus, D., *A remark on Codazzi tensors on constant curvature space*, Lecture Notes Math., **838**, Global Differential Geometry and Global Analysis, Springer-Verlag, New York, 1981.

[6] Gray, A., *Einstein-like manifolds which are not Einstein*, Geom. Dedicata, **7** (1978), 259–280.

[7] Hicks, N. J., *Notes on differential geometry*, Affiliated East West Press Pvt. Ltd., **1969**.

[8] Özgür, C., *$N(k)$-quasi Einstein manifolds satisfying certain conditions*, Chaos, Solitons and Fractals, **38** (2008), 1373–1377.

[9] Özgür, C. and Sular, S., *On $N(k)$-quasi Einstein manifolds satisfying certain conditions*, Bolkan J. Geom. Appl., **13(2)** (2008), 74–79.

[10] Schouten, J. A., *Ricci-Calculus, An introduction to Tensor Analysis and its Geometrical Applications*, Springer-Verlag, Berlin, **1954**.

[11] Özgür, C. and Tripathi, M. M., *On the concircular curvature tensor of an $N(k)$-quasi Einstein manifold*, Math. Pannon., **18(1)** (2007), 95–100.

[12] Patterson, E. M., *Some theorems on Ricci recurrent spaces*, J. London Math. Soc., **27** (1952), 287–295.

[13] Pokhariyal, G. P. and Mishra, R. S., *The curvature tensor and their relativistic significance*, Yokohoma Math. J., **18** (1970), 105–108.

[14] Szabó, Z. I., *Structure theorems on Riemannian spaces satisfying $R(X,Y)\cdot R = 0$, The local version*, J. Diff. Geom., **17** (1982), 531–582.

[15] Tanno, S., *Ricci curvatures of contact Riemannian manifolds*, Tohoku Math. J., **40** (1988), 441–448.

[16] Tripathi, M. M. and Kim, J. S., *On $N(k)$-quasi Einstein manifolds*, Commun. Korean Math. Soc., **22(3)** (2007), 411–417.

A Note On Strongly Pseudoconvex CR-Manifolds And Gray Curvature Conditions

Raluca Mocanu

Romania

Let (M, g, η) a contact metric manifold endowed with the Tanaka-Webster connection $\tilde{\nabla}$ (with the associate curvature tensor $\tilde{R}$).
For this type of manifolds we consider the Gray curvature conditions for the curvature tensor $\tilde{R}$, namely:

$$(\mathbf{G_1^*}): \quad \mathbf{\tilde{R}}(\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{W}) = \mathbf{\tilde{R}}(\mathbf{X}, \mathbf{Y}, \varphi\mathbf{Z}, \varphi\mathbf{W})$$
$$(\mathbf{G_2^*}): \quad \mathbf{\tilde{R}}(\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{W}) = \mathbf{\tilde{R}}(\varphi\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \varphi\mathbf{W}) + \mathbf{\tilde{R}}(\mathbf{X}, \varphi\mathbf{Y}, \mathbf{Z}, \varphi\mathbf{W}) + \mathbf{\tilde{R}}(\mathbf{X}, \mathbf{Y}, \varphi\mathbf{Z}, \varphi\mathbf{W})$$
$$(\mathbf{G_3^*}): \quad \mathbf{\tilde{R}}(\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{W}) = \mathbf{\tilde{R}}(\varphi\mathbf{X}, \varphi\mathbf{Y}, \varphi\mathbf{Z}, \varphi\mathbf{W})$$
(4)

For strictly pseudoconvex CR manifolds the Tanaka-Webster connection coincides with the classical Tanaka connection.
In this note we give examples of manifolds satisfying the $\mathbf{G_i^*}$ conditions ($i \in \{1, 2, 3\}$) and point some remarks on this subject.

Bianchi-Bäcklund Transforms And Dressing Actions, Revisited

Rui Pacheco

University of Beira Interior, Portugal

We characterize Bianchi-Bäcklund transformations of surfaces of positive constant Gauss curvature in terms of dressing actions of the simplest type on the space of harmonic maps.

Holomorphic Extension Of Proper Meromorphic Mappinges On Generic CR-Submanifold

Fatiha Sahraoui

University of Sidi Bel Abbes, Algeria

Forstneric has proved that every proper holomorphic mapping from a ball in $\mathbb{C}^n$ to a ball in $\mathbb{C}^N$ $(N \geqslant n \geqslant 2)$ that is sufficiently smooth on the closure of the ball is in fact a rational mapping. He left open the possibility that such a mapping could be indeterminate at a point of the boundary sphere. Cima and Suffridge have shown that this does not occur, and hence every such mapping extends to be holomorphic in a neighborhood of of the closed ball. After that Chiappari gave some generalization of that result to a real analytic hyper-surphace. The target of my research is to generalize the above results to a generic CR-manifold with arbitrary co-dimension and the result obtained has many applications to study the regularity of the CR-mappings.

On The Osculating Rank Of The Operator R-R* In N-K Manifolds

Javier Seoane-Bascoy

(joint work with L. M. Hervella and A. M. Naveira)

Universidad de Santiago de Compostela, Spain

The osculating rank of the Jacobi operator has been studied by Arias-Marco, Macías, Naveira and Tarrío among others. Following some results due to Alfred Gray for Nearly-Kaehler manifolds. Gray also has showed that the operator $r = R - R^*$ is constant in dimension six and he has indicated an example of dimension ten which does not verify this property. We have proved in a particular case of dimension ten that the osculating rank of de operator r is constant. The problem remains open for an arbitrary Nearly-Kaehler manifold of any dimension.