Duality Principle in pseudo-Riemannian Osserman manifolds

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ABSTRACT

One says that pseudo-Riemannian manifold (M, g) is Osserman if the characteristic polynomial of $\mathcal{J}$ is independent on unit pseudospheres, where $\mathcal{J}_X : Y \mapsto \mathcal{R}(Y, X)X$ is Jacobi operator. Duality principle in Osserman manifolds is property $\mathcal{J}_X(Y) = \lambda Y \Rightarrow \mathcal{J}_Y(X) = \lambda X$ which holds for every Riemannian Osserman manifold.

My work here is the extension of the duality principle in pseudo-Riemannian settings. If $\varepsilon_X = g(X, X)$ is the norm of tangent vector X, the modified duality property should be $\mathcal{J}_X(Y) = \varepsilon_X \lambda Y \Rightarrow \mathcal{J}_Y(X) = \varepsilon_Y \lambda X$, where X and Y are mutually orthogonal nonnull vectors.

If Osserman manifold is diagonalizable (Jacobi operator is diagonalizable for any unit vector) then we can extend domain for X and Y in duality property to the single condition that X is nonnull vector.

If there no exists null eigenvector of Jacobi operator of diagonalizable Osserman manifolds then duality principle holds in pseudo-Riemannian settings. (Specially it is true for Riemannian settings)

The last result is that duality principle holds for every four-dimensional pseudo-Riemannian Osserman manifold.