Osserman and conformally Osserman manifolds

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Abstract.

Let (M, g) be a pseudo–Riemannian manifold and R the curvature tensor. The *Jacobi operator* $\mathcal{J}$ and the *conformal Jacobi operator* $\mathcal{J}_W$ are given by

$$\mathcal{J}(X)Y = R(X, Y)X, \text{ and } \mathcal{J}_W(X)Y = W(X, Y)X,$$

where W stands for the Weyl conformal curvature tensor. We say that (M, g) is pointwise *Osserman* (respectively, *conformal Osserman*) if for every $p \in M$ the Jacobi operator $\mathcal{J}_p$ (respectively, the conformal Jacobi operator) has constant eigenvalues on the unit tangent sphere $S_p = \{x \in T_pM : g_p(x, x) = 1\}$, where T_pM is the space tangent to M on p.

These two concept are closely related since a manifold (M, g) is Osserman if and only if it is Einstein and conformally Osserman. In this talk we review some known results concerning these concepts. Then we concentrate in manifolds whose metric has a special structure; namely, a warped or a twisted product structure. We will give a complete classification of Osserman and conformally Osserman Riemannian manifolds with the structure of a warped product. By means of this approach we analyze the twisted product structure and obtain, as a consequence, that no Osserman Riemannian manifold (that is, no Riemannian two-point-homogeneous manifold) can be written as a twisted product. Moreover, we show how some (but not any) of the results can be extended to pseudo–Riemannian signature.