Invariant submanifolds of a real hypersurface of a complex manifold

We first review some recent characterizations of certain model spaces in complex space forms by studying *n*-dimensional real submanifolds M of Kähler manifolds \tilde{M}^{n+p} whose maximal holomorphic tangent subspace is (n-1)-dimensional, called CR submanifolds of maximal CR dimension. More precisely, besides the submanifold structure, represented by the second fundamental tensor h of M in \tilde{M} , there is another geometric structure, an almost contact metric structure (F, u, U, g), naturally induced from the complex structure of the ambient space. We study certain conditions on the structure F and on h of CR submanifolds of maximal CR dimension in complex space forms and we characterize several important classes of submanifolds in complex space forms.

A typical example of a CR submanifold of maximal CR dimension is a real hypersurface of an almost Hermitian manifold. It is well-known that a real hypersurface is equipped with an almost contact metric structure naturally induced by the almost Hermitian structure of the ambient space. We prove that an odd-dimensional submanifold M of a real hypersurface of a complex manifold which is invariant with respect to this almost contact metric structure is a CR submanifold of maximal CR dimension. Moreover, if the shape operator of a real hypersurface of a non-flat complex space form has two distinct principal curvatures (one of them with multiplicity one), then for all its submanifolds which are invariant with respect to this almost contact metric structure, the second fundamental tensor h and the almost contact structure F satisfy certain condition, mentioned above. We also give the counter example in the case of complex Euclidean case.

This talk is based on joint research with M. Okumura.

Mirjana Djorić, Faculty of Mathematics, University of Belgrade, Studentski trg 16, pb 550, 11000 Belgrade, Serbia, E-mail: mdjoric@matf.bg.ac.yu