Harmonic 2-spheres in Sp(n) and O(n)

R. Pacheco

Departamento de Matemática, Universidade da Beira Interior, Rua Marquês d'Ávila e Bolama 6201-001 Covilhã - Portugal

email: rpacheco@mat.ubi.pt

A map of Riemannian manifolds is *harmonic* if it extremises the energy functional $\int |d\phi|^2 dv$. Harmonic maps from Riemann surfaces are therefore two-dimensional analogues of geodesics. Following Uhlenbeck's seminal work [7], harmonic maps from a simply-connected Riemann surface M to the unitary group $\mathrm{U}(n)$ correspond to certain maps, called *extended solutions*, from M into its loop group $\Omega\mathrm{U}(n)$. When the Fourier series associated to an extended solution has finitely many terms, the corresponding harmonic map is said to be of *finite uniton number.* For example, all harmonic 2-spheres in $\mathrm{U}(n)$ are of this kind. Uhlenbeck also introduced the idea of *uniton factorization* of harmonic maps of finite uniton number into $\mathrm{U}(n)$, and Segal [6] expressed this elegantly by using an infinite dimensional Grassmannian model for the loop group $\Omega\mathrm{U}(n)$. Burstall and Guest extended Uhlenbeck's results to harmonic maps into a general compact Lie group using methods suggested by Morse theory.

In this poster we use Segal's methodology to study harmonic maps of finite uniton number from a compact Riemann surface into the Lie groups $\mathrm{Sp}(n)$ and $\mathrm{O}(n)$. In particular, we give uniton factorizations for such harmonic maps and we give alternative characterizations of harmonic 2-spheres into $\mathbb{H}P^n$, the quaternionic projective space, and $G_2(\mathbb{R}^n)$, the real Grassmannian of 2-dimensional subspaces. These characterizations are compared with those obtained by Bahy-El-Dien and Wood in [2, 3]. In the $\mathbb{H}P^n$ case, our explicit results generalize the work of Aithal [1].

The results we present here concerning harmonic 2-spheres in the symplectic group $\mathrm{Sp}(n)$ were published in [5].

References

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