Sphere Rigidity in the Euclidean Space

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The well-known Alexandrov theorem \[1\] says that embedded hypersurfaces in \(\mathbb{R}^{n+1}\) with constant mean curvature are geodesic spheres. This result is not true for only immersed hypersurfaces. For instance, the so-called Wente’s tori (see \[5\]) are examples of compact surfaces with constant mean curvature in \(\mathbb{R}^3\), which are not geodesic spheres. Other examples of higher genus are known (see \[3\] for instance).

For immersed hypersurfaces of constant mean curvature, an additional assumption is needed. One condition is given by the Hopf theorem \[2\], which says that constant mean curvature spheres immersed in \(\mathbb{R}^{n+1}\) are geodesic spheres.

In this talk, we give a new rigidity theorem for spheres, where we replace the topological assumption by a metric assumption. Precisely, it is easy to see that hypersurfaces of \(\mathbb{R}^{n+1}\) with constant mean curvature and constant scalar curvature are geodesic spheres. This result comes from the fact that a hypersurface of constant mean curvature and constant scalar curvature is totally umbilic. Here, we give a new rigidity result with a weaker assumption on the scalar curvature. Namely, we show

**Théorème 1** Let \((\mathcal{M}^n, g)\) be a compact, connected and oriented Riemannian manifold without boundary isometrically immersed into \(\mathbb{R}^{n+1}\). Let \(h\) be a positive constant. Then, there exists \(\varepsilon > 0\) such that if

1. \(H = h\) and
2. \(|\text{Scal} - s| \leq \varepsilon\),

for some constant \(s\), then \(\mathcal{M}\) is the sphere \(S^n\left(\frac{1}{h}\right)\) with its standard metric.

We derive this theorem from results about almost umbilic hypersurfaces that we will prove before, and based on a previous eignevalue pinching result given in \[4\]. We will give a word about this eigenvalue pinching result.

**References**


