We study the transverse Lusternik-Schnirelmann category of a Riemannian foliation $\mathcal{F}$ on a compact manifold M. We obtain necessary and sufficient conditions for when the transverse category $\mathrm{cat}_{\pitchfork}(M, \mathcal{F})$ is finite. We also introduce a variation on the concept of transverse LS category, the essential transverse category $\mathrm{cat}^{e}_{\pitchfork}(M, \mathcal{F})$, and show that this is finite for every Riemannian foliation. Also, $\mathrm{cat}^{e}_{\pitchfork}(M, \mathcal{F}) = \mathrm{cat}_{\pitchfork}(M, \mathcal{F})$ if $\mathrm{cat}_{\pitchfork}(M, \mathcal{F})$ is finite. A generalization of the Lusternik-Schnirelmann theorem is also given: the essential transverse category $\mathrm{cat}^{e}_{\pitchfork}(M, \mathcal{F})$ is a lower bound for the number of critical leaf closures of a basic C^1-function on M.