Foliations by surfaces of a peculiar class - abstract

Adam Bartoszek(*) and Paweł Walczak(*),(**)
(*) Katedra Geometrii, Wydział Matematyki i Informatyki
Uniwersytet Łódzki, Łódź, Poland
and
(**) Instytut Matematyczny PAN, Warszawa, Poland*

March 28, 2008

It is well known that there exist several obstructions to the existence (on given Riemannian manifolds) of foliations with all the leaves satisfying some geometric properties. For example, by purely topological reason, there exist no codimension-one totally geodesic foliations of round spheres and, by rather dynamical arguments, no such foliations on compact manifolds of negative sectional curvature (in any dimension). Also, there exist no codimension-one totally umbilical foliations of compact manifolds of negative Ricci curvature; in fact, all the foliations of such manifolds are, in a sense, far from being umbilical. Umbilicity is a conformal invariant: if p is an umbilical point of a hypersurface N of a Riemannian manifold (M, g), and $\tilde{g} = e^{2\psi} g$ is a Riemannian metric conformally equivalent to g, then p is umbilical for N on $(M, \tilde{g})$. This is why Rémi Langevin and the second author were searching for other conformally invariant properties providing obstructions to existence of foliations enjoying these properties and have shown that compact 3-dimensional hyperbolic manifolds admit no foliations by *Dupin cyclides* which can be characterized by vanishing of

*E-mail: mak@math.uni.lodz.pl and pawelwal@math.uni.lodz.pl

both conformal principal curvatures. Here, we shall go one step further: after describing local conformal invariants of surfaces and classifying surfaces with all the conformal invariants constant, we shall prove our main result which says that compact hyperbolic 3-manifolds admit no *CCI-foliations*, that is, foliations by surfaces with constant conformal invariants.