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The ICM2006 Section by Section_Lie groups and Lie algebras

"The order of factors does not alter the product" is an expression we hear quite frequently. Nevertheless, many phenomena in life are "non-commutative"; that is, their result depends on the order in which they appear. Imagine for example an aeroplane approaching an airport that receives an instruction to fly ten miles to the north (manoeuvre A) and then turn 180° around the control tower (manoeuvre B). The final position of the aircraft after performing either manoeuvres AB or manoeuvres BA would be completely different.

One of the most fruitful mathematical ideas of the 19th century was the realization that this type of "operation" with movement or displacement, and with more complicated physical transformations, showed a formal resemblance with operations we perform with numbers (except commutativity), which enables them to be studied systematically. It also deals with "non-discrete" phenomena (rotations, transferings, contractions, relativist transformations), which lead us to the use of not only algebraic but also geometrical techniques.

This theory of "continuous transformation groups" was initiated by the Norwegian mathematician Sophus Lie (1842-1899) and is at present an essential tool both in robotics (control theory) and theoretical physics (quantum mechanics). This is the theory which in pure mathematics has enabled all the non-Euclidian geometries to be unified.

Now providing the description of a physical system and its movements or transformations is quite a complex business (imagine a top that spins, lurches and moves). However, it's possible to codify such a system in a few algebraic equations and variables, in a way analogous to plotting the orbit of a planet giving only its velocity and direction at each point instead of its complete trajectory. This algebraic object that codifies and simplifies the description of a continuous transformation group is what we call its "Lie algebra".

At present, the theory of Lie algebras is well established and is used in such varied fields as the study of differential equations (mathematical models) and nuclear spectroscopy. What's more, the so-called Lie superalgebras, related to particle physics, provide a highly active field of research for physicists and mathematicians.

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