Computational methods in Mathematics

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Abstract

Over the last few years, the increasing speed of computers and the development of software that allows the handling of symbolic expressions has provided a very important tool to mathematicians. Symbolic calculus is now used as a source of experimentation and as a way of proving results, specially when the proofs require a great amount of calculations. The software that handles mathematical expressions is no substitute for mathematical thinking; however, it offers the mathematical community an unprecedented tool that speeds up the process of understanding and teaching mathematics. Being capable of taking advantage of these tools is becoming more and more important nowadays both in research and in education.

The aim of this course is to give an introduction to the programming language of *Mathematica* and use this tool explore geometric concepts related to the curvature of surfaces and manifolds. This is the outline of this course:

1. **An introduction to *Mathematica***: The *Mathematica* environment. Numerical and symbolic calculations with *Mathematica*. Manipulating expressions.
2. **Plotting with *Mathematica***: 2D and 3D plots. Visualization of geometric properties of surfaces.
3. **Riemannian geometry with *Mathematica***:

   Basic concepts: coordinates, metrics, curvature. Examples of manifolds.

   Differential operators: gradient, Hessian, divergence, Laplacian, covariant derivatives, Lie derivatives. Change of coordinates.

   Curvature operators and its eigenvalue structure.