Particles, Magnetic Fields, Sigma Models

Manuel Barros

Abstract

The energy functionals depending on curvatures (in the setting of curves, surfaces or, more generally, submanifolds) provide variational models which apply in a wide variety of non linear phenomena. These models and the equations governing their dynamics are important in different subjects, including the following

- Mathematics and Geometry.- Geodesics, Elastic curves, Minimal surfaces, Willmore surfaces...
- **Physics.** Relativistic particles, Landau-Hall problem, Sigma-models and other Field theories, Condensed-matter Physics, High-energy Physics, String theories...
- Biophysics and Biology.- Protein chains, DNA molecules, Interfaces, Vesicles and Biomembranes...

An important aspect of the study of these functionals is that a single example can be of interest in many different contexts. In this sense, they provide amazing connections between apparently unrelated phenomena. This kind of universality is strongly related to the fact that such equations frequently have a deep underlying geometric meaning.

In this mini-course, we will deal with several simple models that apply to different contexts according the above stated philosophy. They have mathematical interest by themselves. However, the corresponding field equations can be applied to describe: the dynamics of relativistic particles, the Landau-Hall problem associated with a Gaussian magnetic field, configurations and solutions of the non linear sigma model with spherical symmetry...