

Book of Abstracts

Symmetry and shape

15 - 18 October 2021

Santiago de Compostela, Spain

<http://xtsunxet.usc.es/symmetry/>

Welcome

According to Felix Klein, geometry is the study of those properties in space that are invariant under a given transformation group. Intuitively, symmetry is the correspondence of shape at every point of a space. An interesting problem in geometry and many physical sciences is to determine the symmetries of a space from its shape.

The aim of this conference is to gather experts in the study of symmetry in differential geometry. The conference will revolve around the study of curvature, homogeneous and symmetric spaces, Riemannian submanifold geometry, and other related topics in Differential Geometry and Geometric Analysis.

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Organization

Organizing committee

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Miguel Domínguez Vázquez, Universidade de Santiago de Compostela, Spain.
Eduardo García Ríó, Universidade de Santiago de Compostela, Spain.

Sponsors

Project PID2019-105138GB-C21 (AEI/FEDER, Spain), Agencia Estatal de Investigación, Spain
Project ED431C 2019/10, Xunta de Galicia, Spain
Project ED431F 2020/04, Xunta de Galicia, Spain
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Schedule

The scientific programme is composed of plenary lectures and short talks. All lectures will take place in Aula Magna of the Faculty of Mathematics.

	Friday	Saturday	Sunday	Monday
9:30				V. Sanmartín
10:00		D. González	G. Solanes	T. Otero
10:30				J. M. Lorenzo
10:50		M. Amann	J. L. Flores	
11:00				
11:10				
12:00		Coffee break	Coffee break	
12:30		L. D. Sperança	J. L. Carmona	
12:50			S. Caeiro	
13:10			E. Pendás	
13:30		Lunch	Lunch	
16:00	A. Kollross	J. M. Manzano	Social activities	
17:00	Coffee break	Coffee break		
17:30	A. Latorre	V. Gimeno		
18:30	A. Rodríguez	J. M. Fernández		
18:50	M. Ferreiro	E. Lucas		
19:10	A. De Arriba	F. Marín		
19:30				

Practical information

Internet connection

You can connect via **eduroam**, if your affiliation supports it and you have your devices correctly configured.

Restaurants

There are several options to have lunch near the conference venue:

- Some restaurants near the campus, such as *Santos*, *Xugo*, *Xantar* or *Altamira*, offer lunch menus for around 10-12 euros.
- Downtown and in the historic center there are many kinds of restaurants. Vegan and vegetarian options can be found, for example, at *Boca a boca*, *Malak Bistro*, *A Tulla* or *The Veggie Carmen*. Many restaurants offer typical Galician food, such as *O Dezaseis* or *María Castaña*.
- The cafeteria-canteens of the Faculty of Mathematics (same building as the conference), of the Escola Técnica Superior de Enxeñaría - ETSE (200m), or the cafeterías Fonseca and Rodríguez-Cadarso (300m) offer menus for 6 euro. Because of students' schedule, it is recommended to have lunch before 13:30.

Abstracts

Invited talks

MAXIMAL ANTIPODAL SETS AND THE TOPOLOGY OF GENERALISED SYMMETRIC SPACES

MANUEL AMANN
(Universität Augsburg, Germany)

After commenting on some properties of (generalised) symmetric spaces we prove several long-standing conjectures by Chen-Nagano on cohomological descriptions of the cardinalities of maximal antipodal sets in symmetric spaces. We actually extend these conjectures to the setting of generalised symmetric spaces of finite abelian p -groups and verify them mostly in this broader context drawing upon techniques from equivariant cohomology theory.

PATH-LIFTING PROPERTIES OF THE EXPONENTIAL MAP WITH APPLICATIONS TO GEODESICS

JOSÉ LUIS FLORES DORADO
(Universidad de Málaga, Spain)

We revisit certain path-lifting and path-continuation properties of abstract maps as described in the work of F. Browder and R. Rheindboldt in 1950-1960s, and apply their elegant theory to exponential maps. We obtain thereby a number of novel results of existence and multiplicity of geodesics joining any two points of a connected affine manifold, as well as causal geodesics connecting any two causally related points on a Lorentzian manifold. These results include a generalization of the well-known Hadamard-Cartan theorem of Riemannian geometry to the affine manifold context, as well as a new version of the so-called Lorentzian Hadamard-Cartan theorem using weaker assumptions than global hyperbolicity and timelike 1-connectedness required in the extant version. If time allows, we will also discuss how this approach also permits to obtain results on the existence of closed timelike geodesics on a Lorentzian manifold.

TOPOGONOV THEOREM AND QUANTUM OPTICS

VICENT GIMENO GARCÍA
(Universitat Jaume I, Spain)

Given a several photons in an optical device, the evolution of the Quantum state is described by a unitary transformation. But not every unitary transformation can be related in such a way with an optical device. In this talk I will explain how to obtain an

approximation for a unitary transformation of the state of a quantum system composed by several photons using an optical device. The key point in the construction of this approximation is the well-known Topogonov Theorem in Riemannian Geometry. This is a joint work with J. C. Garcia-Escartin and J. J. Moyano-Fernández published as “Optimal approximation to unitary quantum operators with linear optics” in Quantum Information Processing (2021) 20:314.

FROM HOMOGENEOUS SPACES TO BIQUOTIENTS: SIMILARITIES AND DIFFERENCES

DAVID GONZÁLEZ ÁLVARO
(Universidad Politécnica de Madrid, Spain)

A biquotient manifold is defined as the quotient of a homogeneous space under a free isometric action. Consequently, the class of biquotients generalizes that of homogeneous spaces. In the first part of this talk we will review some classical and modern results about homogeneous spaces and biquotients, from the point of view of the positivity of various curvature conditions. In the second part we will discuss which vector bundles are, roughly speaking, compatible with the homogeneous or the biquotient structure. The talk will be based on joint work with Miguel Domínguez-Vázquez and Lawrence Mouillé, and independently with Jason DeVito.

TOTALLY GEODESIC SUBMANIFOLDS IN EXCEPTIONAL SYMMETRIC SPACES

ANDREAS KOLLROSS
(Universität Stuttgart, Germany)

Joint work with Alberto Rodríguez-Vázquez. We classify maximal totally geodesic subspaces in exceptional Riemannian symmetric spaces up to isometry. Since the maximal subspaces containing flat factors have been classified by Berndt and Olmos, it suffices to find the semisimple ones. We show that these correspond to subalgebras in the Lie algebra of the isometry group which are maximal among the semisimple subalgebras without compact ideals. To find all such subalgebras of simple real Lie algebras, we use earlier classification results by Dynkin, de Graaf-Marrani and Komrakov.

NILPOTENT LIE ALGEBRAS WITH SMALL CENTER: COMPLEX STRUCTURES AND APPLICATIONS IN PSEUDO-KÄHLER GEOMETRY

ADELA LATORRE LARRODÉ
(Universidad Politécnica de Madrid, Spain)

Complex manifolds are spaces that locally look like \mathbb{C}^n and whose changes of charts are biholomorphic. It is well-known that every n -dimensional complex manifold is a $2n$ -dimensional differentiable manifold. However, deciding when a differentiable manifold M of even dimension $2n$ admits a holomorphic atlas is not straightforward. One approach is given by the Newlander-Nirenberg Theorem, which characterizes complex manifolds as pairs (M, J) where J is an integrable almost-complex structure on M . Nonetheless, the explicit construction of such J 's has proven to be a difficult task.

When M is a nilmanifold, i.e., a compact quotient of a nilpotent Lie group by a lattice, the issue can be slightly simplified when one restricts to the study of *invariant* complex structures. In this case, one can work on the nilpotent Lie algebra \mathfrak{g} underlying M and focus on those complex structures J defined on it. One can then distinguish two types of complex structures, depending on whether the center of \mathfrak{g} admits a J -invariant subspace $\mathfrak{a}_J \neq \{0\}$ or not.

In this talk, we will focus on the case $\mathfrak{a}_J = \{0\}$ and provide the complete classification of pairs (\mathfrak{g}, J) for \mathfrak{g} an 8-dimensional nilpotent Lie algebra. We will then apply our results to the study of pseudo-Kähler geometry, namely, complex manifolds (M, J) endowed with a compatible pseudo-Riemannian metric with closed fundamental form.

EMBEDDED TORI IN $\mathbb{S}^2 \times \mathbb{R}$ WITH CONSTANT MEAN CURVATURE

JOSÉ MIGUEL MANZANO PREGO
(Universidad de Jaén, Spain)

We will describe a 1-parameter family of surfaces in the Riemannian product space $\mathbb{S}^2 \times \mathbb{R}$ with constant mean curvature $H > 0$. Among the constructed surfaces, we find the first non-trivial examples of embedded constant mean curvature tori in $\mathbb{S}^2 \times \mathbb{R}$. They form a continuous deformations from a stack of tangent spheres to a horizontal equivariant cylinder, in the same fashion as Delaunay's unduloids in Euclidean space \mathbb{R}^3 form a continuous deformation from a stack of round spheres to a round cylinder. These non-equivariant tori have constant mean curvature $H > \frac{1}{2}$. This is part of a recent joint work with F. Torralbo (see arXiv:2007.06882).

INHOMOGENEOUS ISOPARAMETRIC HYPERSURFACES IN SYMMETRIC SPACES

VÍCTOR SANMARTÍN LÓPEZ
(Universidad Politécnica de Madrid, Spain)

A hypersurface is said to be isoparametric if it and its nearby equidistant hypersurfaces have constant mean curvature. In this talk, we will see examples of these objects in the context of symmetric spaces together with some classification results. After that, we will construct infinitely many new examples of isoparametric hypersurfaces with novel properties in symmetric spaces of non-compact type and rank greater than two.

LIPSCHITZ-KILLING VALUATIONS IN PSEUDO-RIEMANNIAN MANIFOLDS

GIL SOLANES FARRÉS
(Universidad Autónoma de Barcelona, Spain)

The Lipschitz-Killing invariants discovered by H. Weyl are among the most fundamental quantities that can be assigned to a compact riemannian manifold. Besides Weyl's tube formula, they appear in seemingly unrelated situations such as the kinematic formula of Blaschke-Santaló-Chern and the heat kernel of differential forms. Notably, the Lipschitz-Killing invariants can also be defined on sufficiently nice compact subsets of any riemannian manifold. In this form, they belong to a class of functionals called (smooth) valuations, and they provide a natural extension of the classical quermassintegrals of euclidean convex bodies.

In the talk we will present a joint work with Andreas Bernig and Dmitry Faifman where the Lipschitz-Killing valuations are generalized to the setting of pseudo-riemannian manifolds.

A PROJECTED HOMOGENEOUS RICCI FLOW AND ITS LIMITS

LLOHANN DALLAGNOL SPERANÇA
(Universidade Federal de São Paulo, Brazil)

The Ricci flow was introduced by Hamilton and gained its importance through the years. Of special importance is its limiting behavior and symmetry properties. Taking this into account, here we present a characterization for Gromov-Hausdorff limits of homogeneous spaces. In addition, we present a novel normalization for the homogeneous Ricci flow with natural compactness properties.

As an application, we present a detailed picture of the homogeneous Ricci flow for three-isotropy-summands flag manifolds: phase portraits, basins of attractions, conjugation classes and collapsing phenomena. Moreover, we achieve a full classification of the possible Gromov-Hausdorff limits of the aforementioned lines of flow.

Short talks

CRITICAL METRICS ON THREE-DIMENSIONAL BRINKMANN WAVES

SANDRO CAEIRO

(Universidade de Santiago de Compostela, Spain)

Einstein metrics are critical for the Hilbert-Einstein functional $g \mapsto \int_M \tau \, dvol_g$ when restricted to variations of metrics of constant volume.

Generalizing the functional above, it is natural to construct the functionals given by the L^2 -norm of quadratic invariants of the curvature,

$$\begin{aligned}\mathcal{S} : g &\mapsto \int_M \tau^2 \, dvol_g, \\ \mathcal{T} : g &\mapsto \int_M \|\rho\|^2 \, dvol_g, \\ \mathcal{R} : g &\mapsto \int_M \|R\|^2 \, dvol_g.\end{aligned}$$

In dimension three, the invariants above are related by $\|R\|^2 = 2\|\rho\|^2 - \frac{1}{2}\tau^2$ and hence any quadratic curvature functional is equivalent to \mathcal{S} or

$$\mathcal{F}_t : g \mapsto \int_M \|\rho\|^2 + t\tau^2 \, dvol_g.$$

It follows from the work of Berger that Einstein metrics are critical for all quadratic curvature functionals in dimension three [1]. The main purpose of this talk is to show the existence of many Lorentzian 3-manifolds which are not Einstein but critical for all quadratic curvature functional. In order to construct these examples, we study the existence of critical metrics whose underlying structure is a Brinkmann wave [2].

REFERENCES:

- [1] M. Berger, Quelques formules de variation pour une structure riemannienne, *Ann. Sci. École Norm. Sup.* **3** (1970), no. 4, 285-294. doi.org/10.24033/asens.1194
- [2] M. Brozos-Vázquez, S. Caeiro-Oliveira, E. García-Río, Critical metrics and massive gravity solutions on three-dimensional Brinkmann waves, *Classical and Quantum Gravity*. doi.org/10.1088/1361-6382/ac25e2

ON REDUCTIVE LOCALLY HOMOGENEOUS FEDOSOV MANIFOLDS

JOSÉ LUIS CARMONA JIMÉNEZ

(Universidad Complutense de Madrid, Spain)

A cornerstone for the study of Riemannian homogeneous and locally homogeneous manifolds was laid by W. Ambrose and I. M. Singer in 1958, when they proved the equivalence between homogeneity and the existence of a tensor (homogeneous structure) satisfying certain geometric partial differential equations. We extended this result to reductive homogeneous spaces with some geometric tensors, which do not have to be necessarily pseudo-Riemannian. Therefore, the applications of this result in a non-metric framework shape an ambitious project. We begin this program by studying reductive locally homogeneous Fedosov manifolds, that is, symplectic manifolds equipped with a symplectic and torsion free connection.

This is joint work with M. Castrillón López.

(0,2) MIRROR SYMMETRY ON HOMOGENEOUS HOPF SURFACES

ANDONI DE ARRIBA DE LA HERA
(ICMAT, Spain)

Vertex algebras, introduced by Borchers to prove the Monstrous Moonshine Conjecture, play an important role in many areas of mathematics, such as the representation theory of Kac-Moody algebras and the geometric Langlands correspondence. They have a physical interpretation in 2-dimensional conformal field theory, and have had a strong impact in geometry, first by the construction of the chiral de Rham complex by Malikov-Schechtmann-Vaintrob, used to calculate the elliptic genus by Borisov-Libgober, and more recently by the construction of new superconformal structures on this complex by Heluani-Zabzine among others.

The aim of this talk is to present a new method to construct embeddings of the $N = 2$ superconformal vertex algebra, responsible for mirror symmetry, into the affinization of a quadratic Lie algebra. The new input for the construction is a solution of the “Killing spinor equations” on the quadratic Lie algebra. These equations can be regarded as purely algebraic conditions on the quadratic Lie algebra, but in fact come from geometry and physics, specifically from the approach to special holonomy based on generalized geometry on Courant algebroids. To illustrate this, I will present a geometric example given by a homogeneous Hopf surface. This talk is based on joint work with Luis Álvarez-Cónsul and Mario García-Fernández in arxiv:2012.01851.

ON INAUDIBILITY OF NATURALLY REDUCTIVITY

JOSÉ MANUEL FERNÁNDEZ BARROSO
(Universidad de Extremadura, Spain)

A geometrical property is said to be audible if it can be determined from the eigenvalues of the Laplace-Beltrami operator. In this sense, there are audible properties, for example the volume of the manifold or the total scalar curvature. On the hand, there are properties which have been proved to be inaudible, this is the case of the D’Atri property, and the type \mathcal{A} property (i.e. the Ricci tensor is cyclic parallel).

One of the most important open questions in this topic is about the audibility of the symmetric property. In this sense, the property of a closed Riemannian manifold of being naturally reductive closely generalize the symmetric one. In this talk, it will be shown why the naturally reductive property of a closed Riemannian manifold is in general inaudible.

This is a joint work with Teresa Arias-Marco.

FOUR-DIMENSIONAL PARA-KÄHLER LIE GROUPS

MARÍA FERREIRO SUBRIDO
(Universidade de Santiago de Compostela, Spain)

Lie groups not only constitute a basic tool for producing examples, but also are essential for the classification of homogeneous structures. Left-invariant symplectic and Kähler structures on four-dimensional Lie groups have already been classified by Ovando [5], [6]. The positive definite situation is very rigid since a four-dimensional homogeneous Kähler structure is either symmetric or isometric to the only 3-symmetric space, but the case of neutral signature allows other possibilities with a para-Kähler counterpart. The

description of four-dimensional para-Kähler Lie algebras has been recently approached in different works [1], [2], [7] but it still seems not to be clear.

The purpose of this talk is to approach the description of left-invariant para-Kähler structures on four-dimensional Lie groups and to analyze their geometry together with that of indefinite Kähler Lie groups related to the existence of (para-)Kähler and opposite almost (para-)Kähler structures [4], [3].

REFERENCES:

- [1] G. Calvaruso, A complete classification of four-dimensional para-Kähler Lie algebras, *Complex Manifolds* **2** (2015), 1-10.
- [2] G. Calvaruso, Four-dimensional para-Kähler Lie algebras: classification and geometry, *Houston J. Math.* **41** (2015), 733-748.
- [3] E. Calviño-Louzao, E. García-Río, M. E. Vázquez-Abal, and R. Vázquez-Lorenzo, Local rigidity and nonrigidity of symplectic pairs, *Ann. Global Anal. Geom.* **41** (2012), 241-252.
- [4] A. Fino, Almost Kähler 4-dimensional Lie groups with J-invariant Ricci tensor, *Differential Geom. Appl.* **23** (2005), 26-37.
- [5] G. Ovando, Four dimensional symplectic Lie algebras, *Beiträge Algebra Geom.* **47** (2006), 419-434.
- [6] G. Ovando, Invariant pseudo-Kähler metrics in dimension four, *J. Lie Theory* **16** (2006), 371-391.
- [7] N. K. Smolentsev and I. Y. Shagabudinova, On the classification of left-invariant para-Kähler structures on four-dimensional Lie groups, arXiv:2008.05664 [math.DG].

POLAR HOMOGENEOUS FOLIATIONS ON $SL(3, \mathbb{R})/SO(3)$

JUAN MANUEL LORENZO NAVEIRO
(Universidade de Santiago de Compostela, Spain)

An isometric action of a Lie group over a Riemannian manifold is said to be polar if there exists a submanifold that meets all orbits perpendicularly. Such a submanifold is known as a section, and is totally geodesic. As of today, it is an open problem to classify these actions (up to orbit equivalence) in several families of Riemannian manifolds, mainly symmetric spaces.

The aim of this talk will be to focus on polar actions without singular orbits when the ambient manifold is a symmetric space of noncompact type. In this setting, the existence of a section can be characterized by means of an algebraic criterion [1]. Combining this with the work of Mostow on maximal solvable subalgebras of real semisimple Lie algebras [2], we will arrive at the complete classification of all polar homogeneous foliations on $SL(3, \mathbb{R})/SO(3)$, the space of volume-preserving, self-adjoint and positive definite linear transformations of the Euclidean 3-space.

REFERENCES:

- [1] J. Berndt, J. C. Díaz-Ramos, H. Tamaru: Hyperpolar homogeneous foliations on symmetric spaces of noncompact type, *J. Differential Geom.* **86** (2010), 191-235.
- [2] G. D. Mostow: On maximal subgroups of real Lie groups, *Ann. of Math.* (2) **74** (1961) 503-517.

ON DISCRETE ISOPERIMETRIC TYPE INEQUALITIES

EDUARDO LUCAS MARÍN
(Universidad de Murcia, Spain)

The isoperimetric inequality is one of the oldest and most outstanding results in mathematics, and can be summarized by saying that the Euclidean balls minimize the surface area measure $S(\cdot)$ (Minkowski content) among those compact convex sets with prescribed positive volume $\text{vol}(\cdot)$ (Lebesgue measure). There exist many different versions and extensions of this result, which have led to remarkable consequences in many branches of mathematics. It admits the following “*neighbourhood form*” for any compact convex set $K \subset \mathbb{R}^n$, and all $t \geq 0$,

$$\text{vol}(K + tB_n) \geq \text{vol}(rB_n + tB_n), \quad (1)$$

where $r > 0$ is such that $\text{vol}(rB_n) = \text{vol}(K)$ and B_n denotes the (closed) Euclidean unit ball.

In this talk we discuss and show a discrete analogue of the isoperimetric inequality in its form (1) for the *lattice point enumerator* $G_n(K) = \#(K \cap \mathbb{Z}^n)$ of a bounded subset $K \subset \mathbb{R}^n$: we determine sets minimizing the functional $G_n(K + t[-1, 1]^n)$, for any $t \geq 0$, among those bounded sets K with given positive lattice point enumerator. We also show that this new discrete inequality implies the classical result for compact sets.

This is about a joint work with David Iglesias and Jesús Yepes Nicolás (University of Murcia).

ON GRÜNBAUM TYPE INEQUALITIES

FRANCISCO MARÍN SOLA
(Universidad de Murcia, Spain)

Given a compact set $K \subset \mathbb{R}^n$ of positive volume, if K is convex with centroid at the origin, then, a classical and powerful result by Grünbaum, says that one can find a lower bound for the ratio $\text{vol}(K^-)/\text{vol}(K)$ depending only on the dimension of K , where K^- denotes the intersection of K with a halfspace bounded by a hyperplane passing through its centroid.

In this talk, among other results, we show that fixing the hyperplane H , one can find a sharp lower bound for the ratio $\text{vol}(K^-)/\text{vol}(K)$ depending on the concavity nature of the function that gives the volumes of cross-sections (parallel to H) of K . When K is convex, this inequality recovers the previous result by Grünbaum. To this respect, we also show that the log-concave case is the limit concavity assumption for such a generalization of Grünbaum's inequality.

This is a joint work with Jesús Yepes Nicolás.

COHOMOGENEITY ONE ACTIONS ON IRREDUCIBLE AND REDUCIBLE SYMMETRIC SPACES OF NONCOMPACT TYPE

TOMÁS OTERO CASAL
(Universidade de Santiago de Compostela, Spain)

When studying isometric actions on a given Riemannian manifold, it is natural to ask oneself which ones produce hypersurfaces as their regular orbits: These are the so-called cohomogeneity one actions. This kind of study makes special sense in spaces with a large isometry group, such as the case of symmetric spaces. In the case of irreducible symmetric spaces of noncompact type, Berndt and Tamaru proposed in [1] a general

procedure to classify such actions up to orbit equivalence, proving that they can all be constructed in one of five distinct ways.

The aim of this talk is to report on ongoing work in generalizing Berndt and Tamaru's result to symmetric spaces of noncompact type that might be reducible. With the improvements obtained, we are able to give a list of all possible cohomogeneity one actions on $SL_n(\mathbb{R})/SO_n$ for arbitrary n .

This is a joint work with José Carlos Díaz-Ramos and Miguel Domínguez-Vázquez.

REFERENCES:

- [1] J. Berndt, H. Tamaru, Cohomogeneity one actions on symmetric spaces of noncompact type, *J. Reine Angew. Math.* **683** (2013), 129-159.

CONE STRUCTURES AND THEIR APPLICATIONS TO WAVE PROPAGATION

ENRIQUE PENDÁS RECONDO
(Universidad de Murcia, Spain)

In General Relativity, it is well known that the (infinitesimal) propagation of light rays from a point forms a cone in the spacetime. Each of these cones is symmetric with respect to the origin (since any observer measures the same speed of light in any direction) and can be described by a Lorentz metric. The notion of cone structure then appears when we relax this symmetry condition and allow more general cones, now described by a Lorentz-Finsler metric. Within this framework, Fermat's principle remains valid and the lightlike geodesics satisfy some time-minimizing properties.

These results allow for the application of this geometric framework to describe the propagation of (non-relativistic) waves in the most general situation where there is a time and direction dependence (rheonomic and anisotropic propagation). The cone structure is given in this case by the velocities of the wave, and its fastest trajectories, which form the wavefront, appear as the lightlike geodesics of a suitable Lorentz-Finsler metric.

This talk is based on a joint work with Miguel Ángel Javaloyes and Miguel Sánchez (see [1], [2]).

REFERENCES:

- [1] M. A. Javaloyes, E. Pendás-Recondo and M. Sánchez. Applications of cone structures to the anisotropic rheonomic Huygens' principle. *Nonlinear Analysis* 209, 112337 (2021).
- [2] M. A. Javaloyes and E. Pendás-Recondo. Lightlike hypersurfaces and time-minimizing geodesics in cone structures. arXiv:2109.07969 [math.DG] (2021).

TOTALLY GEODESIC SUBMANIFOLDS AND KÄHLER ANGLE IN SYMMETRIC SPACES

ALBERTO RODRÍGUEZ-VÁZQUEZ
(Universidade de Santiago de Compostela, Spain)

A special class of symmetric spaces where totally geodesic submanifolds can be studied is that of Hermitian symmetric spaces. In these spaces we can use the notion of Kähler angle to measure how a submanifold fails to be complex.

In this talk, I will report on an ongoing work where a method to construct totally geodesic submanifolds with non-trivial constant Kähler angle in non-flat Hermitian symmetric spaces is given.

Participants

1. Manuel Amann, Universität Augsburg, Germany
2. Miguel Brozos Vázquez, Universidade da Coruña, Spain
3. Sandro Caeiro, Universidade de Santiago de Compostela, Spain
4. Esteban Calviño Louzao, Consellería de Educación, Spain
5. José Luis Carmona Jiménez, Universidad Complutense de Madrid, Spain
6. Andoni De Arriba De La Hera, ICMAT, Spain
7. Miguel Domínguez Vázquez, Universidade de Santiago de Compostela, Spain
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