Book of Abstracts

Symmetry and shape

3 - 7 November 2025

Santiago de Compostela, Spain

http://xtsunxet.usc.es/symmetry2025/

Welcome

According to Felix Klein, geometry is the study of those properties in space that are invariant under a given transformation group. Intuitively, symmetry is the correspondence of shape at every point of a space. An interesting problem in geometry and many physical sciences is to determine the symmetries of a space from its shape.

The aim of this conference is to gather experts in the study of symmetry in Differential Geometry, whilst we celebrate Carlos Enrique Olmos' 65th birthday. The conference will revolve around the study of curvature, homogeneous and symmetric spaces, Riemannian submanifold geometry, holonomy, and other related topics in Differential Geometry and Geometric Analysis.

Contents

Organization

Organizing committee

José Carlos Díaz Ramos, Universidade de Santiago de Compostela, Spain
Miguel Domínguez Vázquez, Universidade de Santiago de Compostela, Spain
Eduardo García Río, Universidade de Santiago de Compostela, Spain
Ixchel Dzohara Gutiérrez Rodríguez, Universidade de Vigo, Spain
Alberto Rodríguez Vázquez, Université Libre de Bruxelles, Belgium
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Sponsors

Grant PID2022-138988NB-I00 funded by MICIU/AEI/10.13039/501100011033 and by ERDF, EU
Project ED431C 2023/31, Xunta de Galicia, Spain
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Schedule

The scientific programme is composed of lectures by the main speakers, talks, and short talks. Lectures will provisionally take place in Aula Magna of the *Faculty of Biology*.

There will be a reception at the *Town Hall* of Santiago de Compostela on Wednesday 5th at 18:00.

The conference dinner will take place at *Hotel Palacio del Carmen* on Thursday 6th at 20:00.

Practical information

Internet connection

You can connect via **eduroam**, if your affiliation supports it and you have your devices correctly configured.

Restaurants

There are several options to have lunch near the conference venue:

- Some restaurants near the campus, such as *Sicilia in Bocca*, *Santos*, *Xugo*, *Xantar* or *Altamira*, offer lunch menus for around 12-15 euros.
- Downtown and in the historic center there are many kinds of restaurants. Vegan and vegetarian options can be found, for example, at *Boca a boca*, *Malak Bistro*, *A Tulla* or *The Veggie Carmen*. Many restaurants offer typical Galician food, such as *O Dezaseis* or *María Castaña*.
- The cafeteria-canteens of the Faculty of Mathematics (same building as the conference), of the Escola Técnica Superior de Enxeñaría - ETSE (200m), or the cafeterias Fonseca and Rodríguez-Cadarso (300m) offer menus for 9 euro. Because of students' schedule, it is recommended to have lunch before 13:30.

Abstracts

Invited speakers

The Laplace-Beltrami spectrum on naturally reductive homogeneous spaces

Ilka Agricola
(Philipps-Universität Marburg, Germany)

Calculating the spectrum of the Laplace-Beltrami operator is a fundamental problem in geometric analysis, explicitly solvable mainly for symmetric and normal homogeneous spaces. This talk addresses the broader class of compact naturally reductive homogeneous spaces. We present a Freudenthal-type formula for the Laplacian's spectrum in this setting. This framework provides a powerful tool to analyze how the spectrum behaves under metric deformations, particularly for canonical variations of normal homogeneous metrics. Applications will demonstrate how these methods, particularly when combined with refined branching techniques, lead towards an understanding of the spectrum for 3-(α, δ)-Sasaki manifolds.

This is joint work with Jonas Henkel (Marburg).

Geometry of Vinberg Cones and their Applications

Dmitri Alekseevsky
(Higher School of Modern Mathematics MIPT, Russia)

The talk is based on joint works with Pavel Osipov (Preprint 2025), Vicente Cortés (2021, 2024) and Alessio Marrani and Andrea Spiro (2021, 2023).

We give a short introduction to the Vinberg's matrix theory of homogeneous convex cones (HCC), that describe such cone in terms of T-algebra of rank n matrices of the form $X = ||x_{ij}||$, $x_{ii} \in \mathbb{R}$, $x_{ij} \in V_{ij}$, where V_{ij}, $i, j = 1, \cdots, n$ are Euclidean vector spaces. The matrix multiplication in T-algebra is defined by a system of isometric maps

$$V_{ij} \times V_{jk} \to V_{ik}.$$

The problem of classification of isometric maps, stated by A. Hurwitz (1898), is solved only in the special case, when the dimensions of the two spaces coincide and the problem reduces to description of $\mathbb{Z}_2$ graded Clifford modules.

We define the notion of rank n (special) Vinberg cones C, and describe the rank 3 (special) Vinberg cones, associated to a $\mathbb{Z}_2$ graded Clifford module.

We shortly discuss the differential geometry of rank 3 (special) Vinberg cones C, associated with graded Clifford modules and considered their applications to supergravity.

Then, using Vinberg theory, we reduce the classification of special Vinberg cones of rank n to description of Nil-algebras of Clifford types, that is, associative nilpotent upper triangular algebras of T-matrices.

We reduce the classification of Nil-algebras to enumeration of acyclic directed graphs of diameter 1 and their equipments. The classification of rank 4 special Vinberg cones in terms of corresponding Nil-algebra of Clifford type will be given.

At the end, we shortly discuss the possible application of (special) Vinberg cones.

Outline

1. Homogeneous convex cones (HCC): definition, geometry and applications.
2. E. B. Vinberg theory of HCC: HCC as a cones of positive Hermitian matrices in the space of generalized Hermitian matrices.
3. The problem of classification of isometric maps. Two classes of isometric maps and definition of Vinberg HCC.
4. Definition of Vinberg cones of rank n. Description of rank 3 Vinberg cones, associated to a graded Clifford module. Indefinite case.
5. Applications of rank 3 Vinberg cones to supergravity and to construction of homogeneous special Kähler and quaternionic Kähler manifolds.
6. Bijection between Vinberg cones of rank n and Nil-algebras. (Quasi)Nil-algebras $\mathcal{N}$ and its graph Γ, an equipment of Γ. An approach to classification of Vinberg cones of rank n.
7. Classification of Vinberg cones of rank 4.
8. Applications of Vinberg cones.

Contact hypersurfaces in Hermitian symmetric spaces

Jürgen Berndt
(King's College London, United Kingdom)

Masahiro Kon classified in 1979 the contact hypersurfaces in complex projective spaces. In the talk I will discuss the classification problem for contact hypersurfaces in Hermitian symmetric spaces of compact type.

References:

[1] M. Kon, Pseudo-Einstein real hypersurfaces in complex projective spaces, *J. Differ. Geom.* **14** (1979), 339-354.

A tribute to Carlos

Antonio Di Scala
(Politecnico di Torino, Italy)

This is a space where collaborators and students of Carlos' will share anecdotes and stories. Antonio Di Scala will act as moderator and will show a presentation on his years in FaMAF, Córdoba, Argentina, working with Carlos.

An overview on strong geometries with torsion

Anna Fino
(Università di Torino, Italy)

A strong geometry with torsion is a Riemannian manifold carrying a metric connection with closed skew-symmetric torsion. In the seminar I will first review general properties of metric connections with closed skew-symmetric torsion. Then I will focus on the case of Hermitian manifolds and 7-manifolds endowed with a G_2-structure.

Almost symmetric submanifolds

Claudio Gorodski
(Universidade de São Paulo, Brazil)

We introduce the class of almost symmetric submanifolds of Euclidean space, a close relative of symmetric submanifolds, namely, we require the extrinsic symmetry to have a one-dimensional tangent fixed point set.

In the homogeneous case, we use the rank theorem for submanifolds to reduce the classification to the following cases:

1. most singular orbits in an irreducible s-representation;
2. almost most singular orbits in an irreducible s-representation; and
3. homogeneous submanifolds of codimension three.

We then completely classify these submanifolds.

In the inhomogeneous case, we prove that these submanifolds are of cohomogeneity one and completely describe them as certain unions of parallel symmetric submanifolds in the ambient Euclidean space.

This is joint work with Carlos Olmos.

Rigidity of Riemannian planes without conjugate points

Luis Guijarro
(Universidad Autónoma de Madrid, Spain)

A Riemannian manifold has not conjugate points when the exponential map at any point is a covering map. In the simply connected case, any pair of points is then connected by a single geodesic, making them a natural synthetic replacement for the Euclidean space; it is just enough to replace "lines" by "geodesics". In spite of the aparent simplicity of their geometry, many of the most natural questions remain open, even for low dimensions. In this talk we will look at the 2-dimensional case: Riemannian planes. We will see that, under the extra hypothesis of finite total curvature, a Riemannian plane satisfying Euclid's fisth axiom is necessarily flat; the same conclusion is obtained when assuming the existence of a geodesic foliation with bounded Hausdorff distance among any pair of leaves.

This is joint work with Jian Ge (Beijing Normal University) y Pedro Solórzano (UNAM- Oaxaca).

Einstein manifolds, G_2 geometry and geometric flows

Jason Lotay
(University of Oxford, United Kingdom)

There are infinitely many positive Einstein metrics in dimension 7 and many can be encoded through G_2 geometry. It is natural to ask about the stability of the associated G_2 geometry with respect to natural geometric flows of G_2 structures. I will report on joint works with A. Kennon and J. Stein that address this question, particularly for Einstein metrics with symmetries, demonstrating cases of both stability and instability.

Conformal Curvatures

Paolo Piccione
(Universidade de São Paulo, Brazil)

I will discuss how notions of curvature evolve under conformal changes of metric, with a focus on their interplay with nonlinear elliptic partial differential equations. After reviewing classical and Riemannian curvature, I will turn to conformal geometry, where scalar curvature, Q-curvature, and related invariants are governed by conformally covariant differential operators, such as the Yamabe, Paneitz, and GJMS operators. I will discuss how these operators arise naturally in geometric variational problems, including the Yamabe problem and its higher-order analogues, and highlight recent analytical developments.

Berger-like metrics on sedenion zero divisors

Silvio Reggiani
(Universidad Nacional de Rosario, Argentina)

The sedenion algebra $\mathbb{S}$ is a non-associative, 16-dimensional real algebra obtained from the octonions through the Cayley-Dickson process. It is known that the zero divisors $ZD(\mathbb{S})$ of $\mathbb{S}$—the elements on the sphere with nontrivial annihilator—are diffeomorphic to the Stiefel manifold $V_2(\mathbb{R}^7)$, and that their submanifold geometry is homogeneous under the action of the exceptional compact Lie group G_2. We show that perturbing the metric of $ZD(\mathbb{S})$ along a certain circle fibration, in a way that resembles the construction of Berger spheres from the Hopf fibration, leads to a curve of G_2-invariant metrics of non-negative curvature. This process also yields a new example of an Einstein metric on $V_2(\mathbb{R}^7)$, which can be proven to be G_2-stable.

Totally geodesic submanifolds in the presence of symmetry

Alberto Rodríguez Vázquez
(Université Libre de Bruxelles, Belgium)

Totally geodesic submanifolds play a fundamental role in Riemannian geometry and they are closely related to symmetry. For this reason, their study is particularly rich in highly symmetric settings, such as homogeneous and symmetric spaces, providing deep insights into their underlying geometric structures and properties. In this talk, I will present a broad overview of recent developments in the classification, construction, and applications of totally geodesic submanifolds in these spaces. Emphasizing both classical and new results, I will discuss their connections to Lie group theory and rigidity phenomena, as well as highlight open questions and potential directions for future research.

Gauss linking integrals in symmetric spaces of rank one

Evangelia Samiou
(University of Cyprus, Cyprus)

"A principal problem at the interface of geometria situs and geometria magnitudinis will be to count the intertwinings of two closed or endless curves" (Gauss, note from personal diary, 1833). In that note, Gauss also provides an integral formula for the linking number in 3-dimensional Euclidean space, which was later rediscovered by Maxwell calculating "the algebraic number of turns that one curve embraces the other in the same direction".

In an oriented manifold M the linking number of two disjoint oriented null-homologous submanifolds K, L of complementary dimensions is defined by means of a Poincare dual of one of the submanifolds. Deriving Linking integrals analogous to Gauss' formula then amounts to computing the integral kernel of a right inverse d^{-1} of the Cartan differential d. For the rank one symmetric spaces, we use certain local flows to compute such kernels by integrating Cartan's "magic" homotopy formula.

Symmetry and Nullity in (pseudo) Riemannian homogeneous spaces

Francisco Vittone
(Universidad Nacional de Rosario, Argentina)

The nullity subspace at a point p of a (pseudo-)Riemannian manifold M is defined as the set of tangent vectors that annihilate the curvature tensor. The symmetry subspace consists of all transvections at p that is, the subspace spanned by the directions of parallel Killing vector fields at that point.

If $M = G/H$ is a homogeneous space, both subspaces give rise to invariant distributions, known respectively as the distribution of nullity and the distribution of symmetry. These distributions are closely related, and their existence imposes significant geometric and algebraic restrictions both on M and on the Lie algebra $\mathfrak{g}$ of the presentation group G.

In this talk, we will summarize some recent results on these distributions in the setting of compact Riemannian homogeneous spaces and naturally reductive spaces. We will also discuss the challenges in extending these concepts to the pseudo-Riemannian case and present new results concerning the symmetry and nullity of Lorentzian naturally reductive nilmanifolds.

References:

[1] A. Di Scala, C. Olmos, F. Vittone, Homogeneous Riemannian manifolds with non-trivial nullity, *Transform. Groups* **17** (2022), 31-72.

[2] A. Di Scala, C. Olmos, F. Vittone, The structure of homogeneous Riemannian manifolds with nullity, *Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)* Vol. XXVI (2025), 1049-1069.

[3] B. Luporini, S. Reggiani, F. Vittone, The distribution of symmetry of Lorentzian naturally reductive nilmanifolds, (2025) arXiv:2509.10622.

Long talks

Algebraicity of singular Riemannian foliations

Ricardo Mendes
(University of Oklahoma, USA)

Singular Riemannian foliations are certain partitions of Riemannian manifolds, and the traditional sources of examples are isometric group actions and isoparametric hypersurfaces. When the ambient manifold is a sphere, it has long been known that such examples are, in the appropriate sense, algebraic. In 2018, Lytchak and Radeschi have shown algebraicity for a general singular Riemannian foliation in a sphere. In work in progress, we extend the Lytchak-Radeschi theorem from spheres to any compact normal homogeneous space, a class that includes all compact symmetric spaces. Time-permitting, I'll comment on the ingredients of the proof(s), including results by Heintze-Liu-Olmos

Killing tensors on symmetric spaces

Yuri Nikolayevsky
(La Trobe University, Melbourne, Australia)

I will present some old and new results on geometry and algebra of higher rank Killing tensor fields on Riemannian symmetric space, with a particular focus on the following question: "Is any Killing tensor field on a symmetric space a polynomial in Killing vector fields?" The talk will include some joint results with Vladimir Matveev (Germany) and An Ky Nguyen (Australia).

Classification of cohomogeneity-one actions on reducible symmetric spaces of compact type

Ivan Solonenko
(University of Stuttgart, Germany)

I will report on a joint work in progress with Andreas Kollross, in which we are extending the classification of isometric cohomogeneity-one actions on irreducible symmetric spaces of compact type obtained by Kollross in 1998 to the general reducible case. In order to do so, we first obtain an explicit classification of isometric transitive actions on the aforementioned (reducible) spaces up to subgroup-conjugacy by using the works of Onishchik on decompositions of simple compact Lie algebras. I will restrict to the (less convoluted) case when all the de Rham factors are of non-group type and make comments on the general case.

Short talks

Equivariant Approximation Process of Currents and Spaces with Bounded Curvature

Andrés Ahumada Gómez
(CUNEF Universidad, Spain)

In this talk we introduce actions of a compact Lie group in two regularization processes: in De Rham's approximation process of currents on a smooth manifold by smooth currents, and in a smoothing operator of Riemannian metrics of metric spaces with bounded curvature.

Finite extinction time of a family of homogeneous Ricci flows

Roberto Araujo
(IM PAN, Poland)

The dynamical Alekseevskii conjecture states that if the universal cover of a homogeneous space is not contractible, then every homogeneous Ricci flow solution on it has finite extinction time. Böhm proved the conjecture for compact homogeneous spaces. In this talk, we consider the conjecture for noncompact homogeneous spaces, focusing on the case where the Lie algebra of the transitive isometry group is a semidirect product of a compact Lie algebra and an abelian ideal. We prove the conjecture for a natural family of initial homogeneous metrics satisfying a compatibility condition between the Lie algebra structure and the geometry. As an application, we show that the space of all G-invariant positive scalar curvature metrics on such homogeneous spaces G/H is contractible.

Biquotients that are orbifolds

Nícolas Roberto Ribeiro Caballero
(University of São Paulo, Brazil)

Biquotients are generalizations of homogeneous spaces. The foundations of the theory trace back to Eschenburg's Habilitation [1] in 1984, which was motivated by the representation of an exotic 7-sphere as a biquotient by Gromoll and Meyer [4].

More precisely, let G be a Lie group with a left-invariant metric, and let H be a subgroup of $G \times G$. Denote by H_l and H_r the projections of H onto the left and right factors, respectively, and assume that the metric on G is H_r-invariant. Then the action of H on G,

$$(h_l, h_r) \cdot g = h_l g h_r^{-1},$$

with $(h_l, h_r) \in H$, is isometric. The quotient space of this action is denoted by $G//H$ and is called a *biquotient*. If the action of H on G is (respectively, almost) free, then $G//H$ is a manifold (respectively, an orbifold), and a quotient Riemannian metric is defined such that the projection $G \to G//H$ is a Riemannian submersion. Recall that a Riemannian orbifold of dimension n is a length space locally isometric to the quotient of an n-dimensional Riemannian manifold by a finite group of isometries.

The theorem of Lytchak and Thorbergsson [6] gives a necessary and sufficient condition for a biquotient to be an orbifold. In the context of biquotients, the theorem states that $G//H$ is an orbifold if and only if all slice representations of the H-action on G are polar. In other words, $G//H$ is a biquotient orbifold if and only if the H-action is infinitesimally polar. This characterization has already been used in other classifications of biquotients, among which we mention the biquotients generated by spheres studied by Gorodski and Lytchak [2], and those in compact rank-one symmetric spaces, by Gorodski and Kollross [3].

In this doctoral project, we are classifying infinitesimally polar actions on compact Lie groups of low rank, in order to identify which biquotients admit an orbifold structure.

References:

[1] J. H. Eschenburg, *Freie isometrische Aktionen auf kompakten Lie-Gruppen mit positiv gekrümmten Orbiträumen*, Schriftenr. Math. Inst. Univ. Münster **32** (1984).

[2] C. Gorodski, A. Lytchak, Isometric actions on spheres with an orbifold quotient, *Math. Ann.* **365** (2016), 1041-1067.

[3] C. Gorodski, A. Kollross, Some remarks on polar actions, *Ann. Glob. Anal. Geom.* **49** (2016), 43-58.

[4] D. Gromoll, W. Meyer, An Exotic Sphere With Nonnegative Sectional Curvature, *Ann. of Math.* **100**(2) (1974), 401-406.

[5] A. Kollross, A classification of hyperpolar and cohomogeneity one actions, *Trans. Amer. Math. Soc.* **354**(2) (2002), 571-612.

[6] A. Lytchak, G. Thorbergsson, Curvature explosion in quotients and applications, *J. Differential Geom.* **85**(1) (2010), 117-139.

Index of symmetry of homogeneous spaces diffeomorphic to CROSSes

Ángel Cidre Díaz
(Universidade de Santiago de Compostela, Spain)

Symmetric spaces are among the best-understood classes of Riemannian manifolds and can be viewed as a natural generalization of space forms. The index of symmetry is a geometric invariant that, in a certain sense, measures how far a Riemannian manifold is from being symmetric. This index (and co-index) of symmetry has been mainly studied in the context of compact homogeneous Riemannian manifolds, leading to several general structural results in this setting.

Within compact homogeneous spaces, a particular class for which calculating the symmetry index seems natural is that of homogeneous diffeomorphic spaces to compact-type symmetric spaces. In this talk, we focus specifically on the index of symmetry for homogeneous spaces diffeomorphic to CROSSes (Compact Rank-One Symmetric Spaces).

This is joint work with Carlos Enrique Olmos (Universidad Nacional de Córdoba) and Alberto Rodríguez Vázquez (Université Libre de Bruxelles).

Bicycle tracks with hyperbolic holonomy

Luis Hernández-Lamoneda
(Centro de Investigación en Matemáticas, A.C., México)

We find new necessary and sufficient conditions for the bicycling holonomy of a closed plane curve to be hyperbolic. The manin tool is the "hyperbolic development" of a euclidean plane curve in the hyperbolic plane.

Towards a Ricci flow with surgery from Taub-Bolt to Taub-NUT

John Hughes
(University of Oxford, United Kingdom)

A conjecture of Holzegel, Schmelzer and Warnick states that there is a Ricci flow with surgery connecting the two Ricci flat metrics Taub-Bolt and Taub-NUT. We will present some recent progress towards proving this conjecture.

Polar homogeneous foliations on hyperbolic spaces

Juan Manuel Lorenzo Naveiro
(University of Oklahoma, USA)

An isometric action of a Lie group on a Riemannian manifold is polar if it admits a section—that is, a submanifold that meets every orbit orthogonally. These actions generalize several well-known concepts in geometry, such as the polar coordinate system in the Euclidean plane or the spectral theorem for self-adjoint operators. Because of this, an active research topic has been the classification of polar actions on Riemannian manifolds with a relatively large group of isometries, such as symmetric spaces.

In this talk, I will discuss an ongoing work with José Carlos Díaz-Ramos (Universidade de Santiago de Compostela) in which we aim to classify polar actions without singular orbits (also known as polar homogeneous foliations) on quaternionic hyperbolic spaces and the Cayley hyperbolic plane. This result would close the classification problem for such actions on symmetric spaces of noncompact type and rank one.

Complete cohomogeneity one hypersurfaces into the hyperbolic space

Fernando Manfio
(University of São Paulo, Brazil)

In this talk we will consider isometric immersions $f\colon M^n \to \mathbb{H}^{n+1}$ into hyperbolic space of dimension $n+1$ of a complete Riemannian manifold of dimension n on which a compact connected group of intrinsic isometries acts with principal orbits of codimension one. We give a complete classification if either $n \geq 3$ and M^n is compact or if $n \geq 5$ and the connected components of the flat part of M^n are bounded. This is a joint work with Felippe Guimarães and Carlos Olmos.

Conformal transformations between weighted Einstein manifolds

Diego Mojón Álvarez
(Universidade de Santiago de Compostela, Spain)

A classical problem in Riemannian geometry is that of determining which manifolds admit multiple Einstein representatives of the same conformal class. In the setting of smooth metric measure spaces (Riemannian manifolds endowed with a smooth density function), many Riemannian invariants are naturally modified so that they incorporate information on both the density and the metric. This process gives rise to weighted analogues of Einstein manifolds and conformal classes, allowing for the classical problem to be reformulated in a weighted context.

We analyze smooth metric measure spaces admitting two weighted Einstein structures in the same weighted conformal class, completing partial results found in the literature [2], [3]. We begin by describing the local geometries of these manifolds in terms of certain Einstein and quasi-Einstein warped products, as well as the forms of the density function and the conformal factor relating both structures. Subsequently, we obtain a global classification result under the assumption that one of the underlying metrics is complete. We show that such a manifold is either a weighted analogue of a space form or it belongs to specific families of Einstein or quasi-Einstein warped products. As a consequence, in the compact case, it must be a weighted round sphere.

References:

[1] M. Brozos-Vázquez, E. García-Río, D. Mojón-Álvarez, Conformally weighted Einstein manifolds: the uniqueness problem, arXiv:2504.07860 [math.DG].

[2] J. S. Case, The weighted σ_k-curvature of a smooth metric measure space, *Pacific. J. Math.* **299** (2) (2019), 339-399.

[3] J. S. Case, A Yamabe-type problem on smooth metric measure spaces, *J. Differential Geom.*, **101** (3) (2015), 467-505.

Cohomogeneity one actions on symmetric spaces of mixed type

Tomás Otero Casal
(Universität Münster, Germany)

I will report on ongoing work with Ivan Solonenko and Hiroshi Tamaru where we study cohomogeneity one actions on (products of) symmetric spaces. We show that, with the only exception of a 2-parameter family of diagonal actions, cohomogeneity one actions on a given symmetric space $M_+ \times M_0 \times M_-$ (where M_+ is a symmetric space of compact type, M_0 a euclidean space, and M_- is a symmetric space of noncompact type) decompose as products of actions on each of the individual factors. In particular, we give an explicit description of homogeneous codimension one foliations for any given simply-connected symmetric space.

The Cartan geometry with model the lightlike cone in Lorentz-Minkowski spacetime

Francisco José Palomo
(University of Málaga, Spain)

In this talk, we aim to introduce the problem of studying lightlike hypersurfaces in Lorentzian manifolds. Subsequently, we will examine the viewpoint of the Cartan geometries framework, modeled on the light-cone of Lorentz-Minkowski spacetime, to the understanding and analysis of this problem.

Curvature-adapted orbits of CH1 actions in symmetric spaces of non-compact type

Mario Julián Rodríguez Sánchez de Toca
(Universidade de Santiago de Compostela, Spain)

In Riemannian Geometry, two of the most important operators when studying submanifolds are the shape operator and the Jacobi operator. A hypersurface is said to be curvature-adapted precisely when these two operators have a nice behavior between them, that is, when they diagonalize simultaneously. In this talk we will investigate this property in the presence of symmetry. More precisely, we will provide the classification of curvature-adapted homogeneous hypersurfaces in symmetric spaces of non-compact type.

Nonsmooth Lorentzian geometry

Marta Sálamo Candal
(University of Vienna, Austria)

In recent years, there have been many developments in the field of Lorentzian geometry motivated by the introduction of Lorentzian length spaces by Kunzinger-Sämann (2017). This led to a new formulation of the geometry of general relativity, providing a framework that allows the study of nonsmooth spacetimes, including those that present singularities or with a discrete structure. An important development, in particular, is the characterisation of Ricci curvature bounds by the convexity of an entropic functional, firstly developed by McCann (2018) and Mondino-Suhr (2018) in the smooth setting, and taken into the synthetic setting by Cavalletti-Mondino (2020). In this talk, I will review the basics of this new geometry for Einstein's theory of relativity, and I will discuss some of the research directions that are currently being developed.

Exterior Curvature in Hitchin's Generalised Geometry

Oskar Schiller
(Universität Hamburg, Germany)

Generalised Geometry is concerned with the study of geometrical objects on the generalised tangent bundle $TM \oplus T^*M$ over a manifold M. The basic objects are generalised metrics, generalised connections, and divergence operators. From these, one can obtain other geometrical objects, such as the generalised Riemann tensor.

In this talk, which is based on joint work with Vicente Cortés, we consider an immersed manifold $N \hookrightarrow M$, and investigate how $TN \oplus T^*N$ inherits geometric structures defined on $TM \oplus T^*M$. Assuming N is a hypersurface, we develop the notion of generalised exterior curvature, introducing the generalised second fundamental form and the generalised mean curvature. We present generalised versions of the Gauß-Codazzi equations, and discuss possible applications.

Totally geodesic submanifolds of homogeneous $\mathbb{C}P^{2n+1}$ and $\mathbb{S}^{4n+3}$

Andreas Vermeiren
(KU Leuven, Belgium)

We provide some classification results for totally geodesic submanifolds in complex projective spaces and spheres equipped with an arbitrary $Sp(n)$-invariant homogeneous metric. This talk is based on an ongoing joint work with Michaël Liefsoens and Alberto Rodríguez Vázquez.

Computer-assisted construction of cohomogeneity one Einstein metrics

Qiu Shi Wang
(University of Oxford, United Kingdom)

In this talk, I will present a construction of a new family of $SU(2)$-invariant complete negative Einstein metrics on the complex line bundle $O(-4)$ over $\mathbb{C}P^1$. The proof involves using rigorous numerics to produce an approximate Einstein metric to high precision in a bounded region near the singular orbit, then perturbing it to a genuine Einstein metric using fixed-point methods. At the boundary of this region, we show that the latter metric is sufficiently close to hyperbolic space so that it extends to a complete, asymptotically hyperbolic Einstein metric. Our construction is based on previous work of Buttsworth-Hodgkinson.

Posters

Gordan-Rankin-Cohen operators and weighted densities

Victor Bovdi
(United Arab Emirates University, United Arab Emirates)

We distinguish two classifications of bidifferential operators: between (A) spaces of modular forms and (B) spaces of weighted densities.

(A) The invariant under the projective action of $\mathrm{SL}(2;\mathbb{Z})$ binary differential operators between spaces of modular forms of integer or half-integer weight on the 1-dimensional manifold were found by Gordan (called transvectants), rediscovered and classified by Rankin and Cohen (called brackets), and, in still another context, by Janson and Peetre. The invariant under the algebraic supergroup $\mathrm{OSp}(1|2;\mathbb{Z})$ super modular forms of integer and half-integer weight on $(1|1)$-dimensional superstrings with contact structure were introduced, bidifferential operators between them classified and further studied by Gieres-Theisen [4], Cohen-Manin-Zagier [2], and Gargoubi-Ovsienko [3].

(B) For any complex weights, we classify the analogs of Gordan-Rankin-Cohen (briefly: GRC) binary differential operators between spaces of weighted densities invariant under $\mathfrak{pgl}(2)$. For any complex weights, we classify the analogs of GRC-operators between spaces of weighted densities invariant under the Lie superalgebra $\mathfrak{osp}(1|2)$. In the case of $(1|1)$-dimensional superstring without any additional structure, we also classify the analogs of GRC-operators between spaces of any weighted densities invariant under the Lie superalgebra $\mathfrak{pgl}(1|2)$.

This is a joint research with D. Leites [1].

References:

[1] V. A. Bovdi, D. A. Leites, Gordan-Rankin-Cohen operators and weighted densities, *J. Geom. Phys.* **213** (2025), Paper No. 105497, 6 pp.

[2] P. B. Cohen, Y. Manin, D. Zagier, Automorphic pseudodifferential operators. In: *Algebraic aspects of integrable systems*, Progr. Nonlinear Differential Equations Appl., 26, Birkhäuser Boston, Boston, MA, (1997) 17-47.

[3] H. Gargoubi, V. Ovsienko, Supertansvectants and symplectic geometry, *Int. Math. Res. Not. IMRN* (2008), no. 9, Art. ID rnn021, 19 pp.

[4] F. Gieres, S. Theisen, Superconformally covariant operators and super W-algebras, *J. of Math. Physics* 34 (1993) 5964-85.

Totally geodesic submanifolds of symmetric spaces

Cristina Draper
(Universidad de Málaga, Spain)

The problem of determining the totally geodesic submanifolds of a symmetric space is equivalent to the algebraic problem of determining the subtriples of the corresponding Lie triple system (which can be viewed as the odd part of a $\mathbb{Z}_2$-grading).

In the case of the (Riemannian) symmetric spaces G_2 and $G_2/\mathrm{SO}(4)$, the totally geodesic submanifolds are already determined. Our poster provides very natural descriptions of these submanifolds in terms of associative subalgebras and of Grassmannians, as well as matrix descriptions of the related Lie triple systems for practical use.

Reduction Principles for Proper Coisotropic actions

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We study the core of a proper Lie group action on a smooth manifold, extending a construction known for compact groups. A property of such an action is said to admit a *reduction principle* if it can be understood from the action on the core.

Our focus is on *coisotropic actions* these are proper actions of a Lie group on a symplectic manifold by symplectomorphisms, defined by the condition that the generic orbit is a coisotropic submanifold. We show that coisotropic actions admit a reduction principle and provide several characterizations, extending classical results of Huckleberry and Wurzbacher on compact Kähler manifolds.

Finally, we discuss applications of the core construction to holomorphic isometric actions of compact Lie groups on Kähler manifolds.

Ricci Flow on Five-Dimensional Homogeneous Manifolds

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In 1982, Richard Hamilton introduced a geometric flow derived from an evolution equation involving a Riemannian metric and the respective Ricci tensor. He named this flow by *Ricci flow* and it can be regarded as a non-linear version of the heat equation. This means that the Ricci flow induces a diffusion of the curvature on the manifold. Since the seminal paper of R. Hamilton [3], the Ricci flow has been intensively studied, revealing interesting connections between geometry and topology. A proficuous application of the Ricci flow was provided by Grigori Perelman in order to prove the celebrated Thurston Geometrization Conjecture, of which the Poincaré Conjecture is a consequence.

The aim of this work is to analyze the Ricci flow on five-dimensional homogeneous manifolds in the sense of Thurston (manifolds equipped with an invariant Riemannian metric and admitting compact quotients), as recently classified by Andrew Geng in [2]. Some topics to be explored include the possible pointed limits of the Ricci flow under several normalizations, the existence of ancient or immortal solutions, and the classification of singularity types. Our approach is based on an equivalent flow, namely the *bracket flow*, introduced by Jorge Lauret in [4], [5], which explores the algebraic behavior of homogeneous manifolds. For example, in the case of Lie groups, instead of making the Riemannian metric evolve, we fix it and evolve the Lie bracket in the Lie algebra. The case of homogeneous spaces is more delicate and requires a reductive decomposition. Technically, this leads to a dynamical system defined on a subset of the algebraic variety of Lie brackets. In the literature, the case of dimensions 3 and 4 was analyzed by Jorge Lauret and Romina Arroyo in [5], [1], respectively.

We would like to thank FAPESP for financial support.

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Spheres with parallel mean curvature in $\mathbb{S}^2 \times \mathbb{H}^2$

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In Riemannian products of two real space forms, it is known that surfaces with parallel mean curvature vector admit a holomorphic quadratic differential. We consider the product of a sphere and a hyperbolic plane with opposite sectional curvature. Within this product space, we study the parallel mean curvature surfaces for which the differential vanishes. In particular, this includes the surfaces which have the topology of a sphere.

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